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THE ORIGIN OF THE NEBULAR LINES AND THE STRUCTURE OF THE PLANETARY NEBULAE

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ABSTRACT

Identification of nebular lines.—Eight of the strongest nebular lines are classified as due to electron jumps from metastable states in N_{II}, O_{II} and O_{III}. Several of the weaker lines are identified with recently discovered lines in the spectrum of highly ionized oxygen and nitrogen.

Behavior of lines in nebulae.—The lines thus identified are shown to behave in various nebulae in a way consistent with the foregoing classifications. A similar study of the few lines yet unknown makes it possible to estimate the stage of ionization from which they arise.

Structure of the planetary nebulae.—On the basis of the foregoing identifications, the relative sizes and intensities of the monochromatic images of the planetary nebulae are explained by an extension and modification of the ideas developed by Zanstra for hydrogen in the diffuse nebulae.

I IDENTIFICATION OF NEBULAR LINES

In the spectrum of the gaseous nebulae several very strong lines are observed that have not been reproduced in the laboratory. At first these lines were ascribed to some unknown element "nebulium." From the character of the other elements, H, He, C, N, and O, known to be present in the nebulae it was quite certain that these lines must be emitted by some element of low atomic weight. The development of present ideas concerning atomic structure, however, leaves no place for an unknown element of low atomic weight. Further, Wright's studies[1] of the relative intensities of these lines in a large

[1] *Publications of the Lick Observatory*, **13**, 193, 1918; *Publications of the Astronomical Society of the Pacific*, **32**, 63, 1920.

number of nebulae show that the various lines behave in such widely different ways that they can hardly all come from the same element.

All these considerations lead to the conclusion, expressed by H. N. Russell, that "it is now practically certain that they must be due not to atoms of unknown kinds but to atoms of known kinds shining under unfamiliar conditions."[1] This unfamiliar condition Russell suggests to be low density.[2]

One type of line, which would be possible only under conditions of very low density, is that produced by an electron jump from a metastable state. A metastable state may be considered to be one from which jumps are very improbable, i.e., one whose mean life before spontaneous emission is very long. Under laboratory conditions the mean time between impacts of a given atom with other atoms or with the walls is, even in the most extreme cases, only 1/1000 second. Consequently, an atom in a metastable state will, in general, be dropped down to a lower state by a collision of the second kind long before it will be able to return directly with the emission of radiation. In the nebulae, however, where the mean time between impacts is variously estimated at from 10^4 to 10^7 seconds, such atoms will return spontaneously and radiate a line with the frequency corresponding to the difference in energy between the metastable state and the final state. Since the probability of emission of these lines is very small, the probability of their absorption must also be small. Thus these lines should not be observed in absorption.

As stated above, *H*, *He*, *C*, *N*, and O are the only elements known to exist in the nebulae. C_{I}, N_{I}, N_{II}, O_{I}, O_{II}, and O_{III} are the only ions of these elements that have metastable states so placed that jumps from them would give rise to lines in the region of wave-lengths that is observable in nebulae. Since the low stages of ionization of these elements are not observed in the nebulae, C_{I}, N_{I}, and O_{I} can at once be eliminated. In a four-valence-electron system such as N_{II} and O_{III} the normal configuration of two 2s- and two 2p-electrons is characterized by ^{3}P-, ^{1}D-, ^{1}S-terms. Of these, $^3P_0$ is the stable ground state

[1] Russell, Dugan, and Stewart, *Astronomy*, 1927, p. 837.

[2] Russell further states the reason why new lines might be emitted in a gas of low density as follows: "This would happen, for example, if it took a relatively long time (as atomic events go) for an atom to get into the right state to emit them, and if a collision with another atom in this interval prevented the completion of the process."

while the rest are metastable since any jump from them involves zero change in the azimuthal quantum number. In a five-electron system such as O_{II} the normal configuration of two 2s- and three 2p-electrons has ^4S-, ^2D-, ^2P-terms of which ^4S is the stable state and ^2D and ^2P are metastable.

The only cases where the differences between these terms are accurately known[1] are $^1D-{}^1S$ of O_{III} and $^2D-{}^2P$ of O_{II}. These are 22,916 and 13,646 frequency units which correspond to wave-lengths of 4362.54 and 7326.2 A, respectively. Two of the strongest nebular lines are found at 4363.21 and 7325 A. The deviation between calculated and observed values correspond to about 3 frequency units. Since the foregoing wave-lengths were calculated from the difference in frequencies of lines in the region between 500 and 800 A, this corresponds to an error of only about .01 A in these lines. The group at 7325 A should have three components with an extreme separation of about 10 A. As the only observations of this line were made with an instrument having a dispersion of 600 A per millimeter, it is not surprising that the line was not resolved.

The $^4S-{}^2D$-group in O_{II} can be predicted roughly from the difference between the term values. As no intercombinations between quartets and doublets have been observed, this difference depends solely on the independent adjustment of the terms by series formulae and therefore is only approximate. This difference predicts a pair at 27,157 and 27,175 frequency units, or 3681.25 and 3678.81 A. The two strongest ultra-violet nebular lines are 3728.91 and 3726.16 A. In view of the uncertainties mentioned above, the agreement in position and separation are both satisfactory.

Two other nebular pairs are at 5006.84, 4958.91 and 6583.6, 6548.1 A. The frequency separation of these are 193 and 82.3 cm^{-1}, respectively, while the separation of $^3P_1-{}^3P_2$ in O_{III} is 192 cm^{-1} and in N_{II} 82.7 cm^{-1}. This quite certainly identifies these pairs as $^3P_2-{}^1D_2$ and $^3P_1-{}^1D_2$ of O_{III} and N_{II}. In the case of N_{II} a check on this identification is possible as A. Fowler and L. J. Freeman[2] have found intercombinations between singlets and triplets which enable them to fix certain of the singlet levels relative to the triplet levels. The fore-

[1] Bowen, *Physical Review*, **29**, 231, 1927.

[2] *Proceedings of the Royal Society*, **114**, 662, 1927.

going identification of the pair of nebular lines as $^{3}\mathrm{P}-{}^{1}\mathrm{D}$ enables one to fix the position of $^{1}\mathrm{D}$ relative to these same triplet levels and then to calculate accurately the frequency of the combinations between the $^{1}\mathrm{D}$- level and certain of the singlet levels found by Fowler and Freeman. These calculations predict that the combination with the $^{1}\mathrm{P}$-level of the $s^2p \cdot s$-configuration should give rise to a line at 746.98 A and with the $^{1}\mathrm{D}$ of $s^2p \cdot d$ to a line at 582.15 A. Two strong nitrogen lines that have not previously been classified occur on plates obtained in this laboratory at 746.97 and 582.16 A.

Of the other possible jumps from the metastable state of these ions $^{4}\mathrm{S}-{}^{2}\mathrm{P}$ of O_{II}, $^{3}\mathrm{P}_1-{}^{1}\mathrm{S}$ of O_{III}, and probably $^{3}\mathrm{P}_1-{}^{1}\mathrm{S}$ of N_{II} fall below 3300 A where they cannot be observed in the nebulae. $^{1}\mathrm{D}-{}^{1}\mathrm{S}$ of N_{II} should occur with a wave-length somewhat greater than 5500 A where photographic observations are difficult except in the case of the strongest line. Thus all of the lines expected on this hypothesis are found, and in so doing all but two or three of the strong nebular lines are explained.[1]

These nebular lines constitute the first direct experimental evidence for the idea, expressed above, that metastable states are not absolutely metastable but are states whose mean life is long before spontaneous radiation begins, the electron being able to return from them after the proper lapse of time even under conditions where the atom cannot be affected by surrounding atoms.

[1] Since the foregoing article was written Croze and Mihul (*Comptes rendus*, **185**, 702, 1927) and Russell (*Physical Review*, in print) have independently found intercombination lines in O_{II}. Russell, using his own identifications, and Fowler (*Nature*, **120**, 617, 1927), the identifications of Croze and Mihul, have shown that the calculated frequencies of the $^{4}S-{}^{2}D$ lines differ from those of the nebular pair at 3726, 3729, by only 7 frequency units. This adds further confirmation to the identification made above.

Fowler (*op. cit.*, p. 582, 1927) has also found intercombinations in O_{III} and used them to test the identification of the $^{3}P-{}^{1}D$ lines with the 5007, 4959 nebular pair, which was made above on the basis of their doublet separation. Using a somewhat better determination of the 374 A line, viz., $\lambda = 374.03$ A and $\nu = 267{,}358$ cm^{-1} than was available to Fowler, the agreement in frequency is well within the rather large experimental error in this extreme ultra-violet line.

Until a higher order of accuracy is obtainable in the extreme ultra-violet, this is as far as comparisons of the observed and calculated positions of these lines can be carried, since the identifications of all of the other lines had already been confirmed by direct comparisons given in this and also in the preceding preliminary articles (Bowen, *op. cit.*, p. 473, 1927, and *Publications of the Astronomical Society of the Pacific*, **39**, 295, 1927).

TABLE I

SERIES CLASSIFICATION OF NEBULAR LINES

λ I.A.	Source	Series Designation*	Excitation Potential†
3313	O_{III}	$3k^3P_1-3m^3S$	36.72
3342	O_{III}	$3k^3P_2-3m^3S$	36.72
3346	O_{IV}?	$3m^2P-3n^2D$	51.76
3426.2	N_{IV}?	3^3S-3^3P	47.
3445	O_{III}	$3m^3P_2-3n^3P_2$	40.66
3704	H_ξ, He_I	2^3P-7^3D	13.49, 24.22
3712	H_ν		13.49
3722	H_μ		13.48
3726.12	O_{II}	$a^4S-a^2D_2$	3.31
3728.91	O_{II}	$a^4S-a^2D_3$	3.31
3734	H_λ		13.47
3750	H_κ		13.45
3759	O_{III}	$3k^3P_2-3m^3D_3$	36.19
3771	H_ι		13.43
3798	H_θ		13.41
3820	He_I	3^3P-6^3D	24.11
3835.5	H_η		13.38
3840.2			
3868.74			
3888.96	H_ζ, He_I	2^3S-3^3P	13.33, 22.92
3935	Ca_{II}?	$4^2S-4^2P_2$	3.14
3964.8	He_I	2^1S-4^1P	23.65
3967.51			
3970.08	H_ϵ		13.27
4009	He_I	2^1P-7^1D	24.22
4026.2	He_{II}, He_I	2^3P-5^3D	53.89, 23.95
4064			
4068.62			
4076.22	O_{II}?	$3m^4D_4-3n^4F_5$	28.60
4097.3	N_{III}	$3k^2S-3m^2P_2$	30.35
4101.74	H_δ, N_{III}	$3k^2S-3m^2P_1$	13.17, 30.34
4120.6	He_I	2^3P-5^3S	23.88
4144.0	He_I	2^1P-6^1D	24.12
4200	He_{II}		53.76
4267.1	C_{II}	$3n^2D-4^2F$	20.87
4340.46	H_γ		13.00
4353			
4363.21	O_{III}	a^1D-a^1S	5.33
4388.0	He_I	2^1P-5^1D	23.95
4416	O_{II}?	$3k^2P_{1,2}-3m^2D_{2,3}$	26.18
4471.54	He_I	2^3P-4^3D	23.64
4541.4	He_{II}		53.54
4571.5			
4634.1	N_{III}	$3m^2P_1-3n^2D_2$	33.01

TABLE I—*Continued*

λ I.A.	Source	Series Designation*	Excitation Potential†
4640.9	N_{III}	$3m^2P_2-3n^2D_3$	33.01
4649.2	C_{III}	3^3S-3^3P	30.1
4658.2	....	....	....
4685.76	He_{II}	....	50.82
4711.4	....	....	....
4712.6	He_I	2^3P-4^3S	23.50
4725.5	....	....	....
4740.2	....	....	....
4861.32	H_β	....	12.70
4922.2	He_I	2^1P-4^1D	23.64
4958.91	O_{III}	$a^3P_1-a^1D_2$	2.50
5006.84	O_{III}	$a^3P_2-a^1D_2$	2.50
5017	He_I	2^1S-3^1P	23.00
5411.3	He_{II}	....	53.10
5655	....	....	....
5737	....	....	....
5754.8	N II	$p^2\,{}^1S_0-p^2\,{}^1D_2$	....
5875.7	He_I	2^3P-3^3D	22.98
6302	....	....	....
6313	....	....	....
6364	....	....	....
6548.1	N_{II}	$a^3P_1-a^1D_2$	1.89
6562.79	H_α	....	12.04
6583.6	N_{II}	$a^3P_2-a^1D_2$	1.89
6677	He_I	2^1P-3^1D	22.98
6730	....	....	....
7009	....	....	....
7065	He_I	2^3P-3^3S	22.63
7138	....	....	....
7325	O_{II}	a^2D-a^2P	5.00

* The electron configurations are indicated as follows: Terms arising from the most stable configuration of (2) 2*s*- and (n−2) 2p-electrons ("n" is the number of valence electrons remaining in the atom) are indicated by "a"; from (2) 2s, (n−3) 2p-electrons, and an excited s-electron by "k"; from (2) 2s, (n−3) 2p-electrons, and an excited p-electron by "m"; from (2) 2s, (n−3) 2p-electrons, and an excited d-electron by "n." The numeral preceding these letters indicates the total quantum number of the excited electron. In the case of one-and-two-valence-electron systems these letters are omitted since for all cases here considered the type of the term is the same as the type of the orbit of the excited electron.

† The excitation potential given represents the energy necessary to place the ion or atom in a state where it can radiate the line under consideration.

In addition to the lines identified as being produced by the foregoing mechanism, certain of the remaining weak lines can also be classified by direct comparison with the lines found in recent work in the laboratory on high stages of ionization. Thus in the data of Mihul[1] on O_{III} the five strongest lines in the region above 3300 A are 3312.35, 3340.78, 3444.15, 3754.65, and 3759.83 A. Of these, 3754.65

[1] Mihul, *op. cit.*, **183**, 1035, 1926; **184**, 89, 874, 1055, 1927.

A is probably obscured by H_κ, while the remainder correspond to 3313, 3342, 3445, and 3759 in Wright's list of nebular lines. Likewise, the nebular lines at 4097.3, 4634.1, and 4640.9 correspond to lines[1] that Fowler has identified in N_{III}. Strong O_{II} lines may correspond to 4076.22 and 4416 A. That the lines mentioned in this paragraph are regular series lines rather than lines due to jumps from metastable states is confirmed by the observation that most of these lines are also present in the spectrum of the nuclei of the planetary nebulae and in other very hot stars.

Table I is a summary of all available data on the origin, series classification, and excitation potential of all lines given by Wright.

II. BEHAVIOR OF LINES IN NEBULAE

In his Table 13 Wright[2] gives the intensities of the various nebular lines as they occur in thirty-six different nebulae. These intensities are adjusted for each nebula to make the sum of the intensities of H_β and H_γ equal to 100. Consequently, his intensity for any other line represents the intensity of that line relative to the hydrogen lines. In Table II of this paper the data for all lines arising from a given ion have been grouped together and these groups arranged in order of the ionization potentials of the ions. The order of the nebulae has also been changed so that in the present table they stand in the order of the intensity with which 5007 and 4959 appear in them. Since these lines are due to O_{III}, which has an ionization potential four times that of hydrogen, their intensity relative to the hydrogen lines can be taken as a measure of the degree of ionization present in any particular nebula. Therefore, this arrangement of the nebulae in the order of intensity of the 5007 and 4959 lines is also an arrangement in order of intensity of ionization in them.

Even a superficial inspection of this table shows that the lines identified as O_{III} behave very much like the He_{II} lines, as would be expected from the approximate equality of their ionization potentials (54.8 and 54.18 volts). The lines identified as O_{II} show characteristics similar to other elements of lower ionization potential. The data for the N_{II} lines are very incomplete (the few figures given are

[1] Bowen, *Physical Review*, **29**, 231, 1927.

[2] *Loc. cit.*

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TABLE II

Intensity of Spectral Lines in Nebulae

Pl

Ion I.P.	N G C λ	6741	6790	6886	7662	2440	7027	6884	I C 5117	7026	6818	I C 1747	3242	I C 2165	I C 351	6891	I C 4634	1535	2392
H 13.54	4861	63	53	57	48	49	57	56	60	57	51	50	50	53	53	59	50	52	50
	4340	37	47	43	52	51	43	44	39	43	49	50	51	47	47	41	50	48	50
	4102	20	28	21	29	20	24	28	17	28	22	tr	34	26	26	23	27	25	29
	3970	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3835	....	9	10	5	....	8	6	....	....	....	....	....	5	10	....	6	....	....
	3798	....	6	....	4	....	5	4	....	....	....	....	5	4	5	....	5	....	....
	3771	....	4	....	....	....	4	....	....	....	....	....	....	....	....	....	4	....	....
	3750	....	3	....	....	....	3	....	....	....	....	....	....	....	....	....	....	....	....
	3734	....	3	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3712	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
*He*ɪ 24.41	4922	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	4713	5	3	10	6	....	9	6	....	....	....	....	5	8	....	....	....	5	tr
	4471	....	13	7	4	....	9	10	....	10	....	....	8	10	....	5	9	5	....
	4388	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	4144	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	4121	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	4009	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3965	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3820	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....
*C*ɪɪ 24.29	4267	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
*N*ɪɪ 29.56	6584	....	....	....	....	....	19	....	....	....	....	....	....	....	....	....	....	....	....
	6548	....	....	....	....	....	6	....	....	....	....	....	....	....	....	....	....	....	....
*O*ɪɪ 35.00	3729, 3726	46	11	40	13	52	12	17	....	28	34	tr	16	19	....	25	13	....	34
*C*ɪɪɪ 45.5	4649	....	....	....	....	....	5	....	....	....	....	....	....	....	....	....	....	....	....
*N*ɪɪɪ 47.2	4641	5	....	5	4	....	10	6	....	....	....	....	5	10	....	....	....	....	....
	4634	....	....	....	1	....	6	....	....	....	....	....	....	....	....	....	....	....	....
	4097	....	....	....	....	....	2	....	....	....	....	....	....	....	....	....	....	....	....
*He*ɪɪ 54.18	4686	46	3	49	59	62	43	22	....	20	60	....	49	55	58	tr	....	25	70
	4541	....	....	....	4	....	7	....	....	....	....	....	5	....	....	....	....	....	....
	4200	....	....	....	3	....	3	....	....	....	....	....	5	....	....	....	....	....	....
*O*ɪɪɪ 54.8	3759	....	....	....	....	....	4	....	....	....	....	....	....	....	....	....	....	....	....
	3445	....	....	4	13	4	8	....	....	....	tr	....	23	4	17	....	....	....	....
	3342	....	....	....	5	....	....	....	....	....	....	....	5	....	....	....	....	....	....
	3313	....	....	....	2	....	1	....	....	....	....	....	2	....	....	....	....	....	....
	5007	370	360	330	360	280	300	286	....	....	....	200	184	194	182	194	190	155	170
	4959	170	180	180	145	175	151	160	140	115	112	110	123	108	110	86	82	109	85
	4362	20	26	26	15	20	30	22	26	....	15	....	15	27	18	....	8	20	37
Un-known	3346	....	....	....	2	5	8	....	....	....	....	....	....	....	....	....	....	....	11
	3426	24	....	17	6	50	20	....	....	....	16	....	4	10	....	....	....	....	35
	3869	62	68	57	65	62	55	62	48	55	47	60	71	51	55	30	46	63	60
	3967	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	4064	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....	....	....	....
	4069	....	3	tr	....	....	9	....	....	....	....	....	....	....	....	....	....	....	....
	4076	....	....	....	....	....	4	....	....	....	....	....	....	....	....	....	....	....	....
	4571	....	....	....	....	....	5	....	....	....	....	....	....	....	....	....	....	....	....
	4658	....	....	....	....	....	5	....	....	....	....	....	....	....	....	....	....	....	....
	4726	....	....	....	....	....	5	....	....	....	....	....	....	....	....	....	....	....	....
	4740	5	3	10	6	....	15	6	....	....	....	....	6	8	....	....	....	10	tr

Ia Ic

Pa

Pe Pb Pc P7 Pf Pd

TABLE II—*Continued*

Ion I.P.	NGC λ	IC 3568	IC 4776	6807	6803	IC 5217	7009	IC 4846	6879	6572	IC 4593	6833	6543	IC 2149	IC 4997	IC 418	40	6210	6826
H 13.54	4861	46	53	55	52	53	50	49	56	50	53	49	51	50	48	51	51	50	49
	4340	53	47	45	48	47	50	51	44	50	47	51	49	50	52	49	49	50	51
	4102	34	26	22	24	30	30	30	20	31	30	26	30	37	30	29	25	28	30
	3970	....	....	....	....	....	....	....	....	....	....	....	....	....	....	20	15	10	....
	3835	....	8	5	....	10	6	....	....	14	....	5	11	10	10	17	....	7	....
	3798	....	6	3	....	7	5	....	....	10	....	4	8	8	9	12	....	5	....
	3771	....	5	2	....	5	....	....	....	8	....	....	6	6	8	9	....	4	....
	3750	....	4	....	....	5	....	....	....	5	....	....	....	4	7	8	....	3	....
	3734	....	....	....	....	....	....	....	....	....	....	....	....	....	6	....	....	....	....
	3712	....	....	....	....	....	....	....	....	....	....	....	....	....	3	....	....	....	....
He$_{I}$ 24.41	4922	....	....	....	....	....	....	....	....	2	....	....	....	....	....	....	....	....	....
	4713	....	....	....	....	6	8	....	....	3	....	....	3	....	....	....	....	....	2
	4471	....	10	6	10	13	10	....	....	12	tr	10	11	16	16	10	....	9	10
	4388	....	....	....	....	....	....	....	....	2	....	....	2	....	....	1	....	....	3
	4144	....	....	....	....	....	....	....	....	2	....	....	....	....	....	....	....	....	....
	4121	....	....	....	....	....	....	....	....	2	....	....	....	....	....	....	....	....	....
	4009	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....
	3965	....	....	....	....	....	....	....	....	2	....	....	2	....	....	....	....	....	....
	3820	....	3	....	....	....	....	....	....	3	....	....	....	....	5	....	....	....	....
C$_{II}$ 24.29	4267	....	....	....	....	....	3	....	....	3	....	....	....	....	....	....	....	....	....
N$_{II}$ 29.56	6584	....	....	....	....	....	....	....	....	23	....	....	8	....	....	....	....	....	....
	6548	....	....	....	....	....	....	....	....	8	....	....	....	....	....	....	....	....	....
O$_{II}$ 35.00	3729, 3726	....	28	12	20	15	20	21	....	21	45	tr	40	55	20	85	97	30	22
C$_{III}$ 45.5	4649	....	....	....	....	....	....	....	....	2	....	....	....	....	....	....	....	....	....
N$_{III}$ 47.2	4641	....	....	....	....	5	8	....	....	2	....	....	3	....	....	....	....	....	....
	4634	....	....	....	....	....	2	....	....	1	....	....	....	....	....	....	....	....	....
	4097	....	....	....	....	....	5	....	....	2	....	....	4	....	....	....	....	....	5
He$_{II}$ 54.18	4686	....	....	....	....	12	28	....	....	....	....	....	....	....	....	....	....	....	....
	4541	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	4200	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
O$_{III}$ 54.8	3759	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3445	....	....	....	....	tr	10	....	....	....	....	....	....	....	....	....	....	....	....
	3342	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3313	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	5007	160	....	....	130	....	130	126	125	120	107	116	92	79	47	33	tr	....	....
	4959	92	77	77	100	75	85	78	77	75	74	53	58	49	30	22	tr	....	....
	4363	tr	14	17	12	16	15	7	10	14	....	16	3	....	57	1	....	10	8
Unknown.	3346	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3426	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	3869	48	47	43	59	55	70	59	38	50	20	45	37	26	62	....	....	55	40
	3967	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	30	....
	4064	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....
	4069	....	6	....	....	....	....	....	....	5	....	....	....	tr	9	10	....	3	....
	4076	....	....	....	tr	....	....	....	....	2	....	....	....	....	6	2	....	....	....
	4571	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....
	4658	....	....	....	....	....	....	....	....	1	....	....	....	....	....	....	....	....	....
	4726	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
	4740	....	....	....	....	6	8	....	....	3	....	....	....	....	....	....	....	....	....

Ib IIa IIb III

taken from another table) but seem to be consistent with their identification.

The lines still unidentified are included at the bottom of the table. A study of the intensities at once indicates that 3868.74 (and

3967.51 which Wright classifies with it) come from an ion with a relatively low ionization potential. On the other hand, 3426.2 and 3346 have the sharpest increase of intensity at the left of the table of any of the lines. This was recognized by Wright when he used the condition that 3426.2 should be stronger than 3445 of $O_{\rm III}$ as the criterion for the first or I*a* class in his system of nebular classification. This necessitates that these lines come from ions whose ionization potentials are greater than that of $O_{\rm III}$. A rough prediction from the

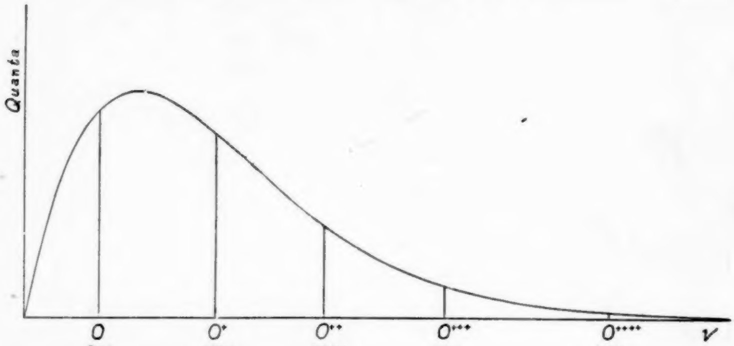

FIG. 1.—Distribution in frequency of quanta emitted by a black body at 150,000°K.

lines in $Be_{\rm I}$, $B_{\rm II}$, and $C_{\rm III}$ indicates that the only strong line of $N_{\rm IV}$ in the region between 3300 and 7000 A is 3^3S-3^3P, which is predicted at 3460 ± 50 A. A similar prediction from $B_{\rm I}$, $C_{\rm II}$, and $N_{\rm III}$ fixes 3^2P-3^2D of $O_{\rm IV}$ at 3440 ± 100. This is the only line of $O_{\rm IV}$ to be expected in this range. This makes identification of 3426.2 as $N_{\rm IV}$ and 3346 as $O_{\rm IV}$ quite probable, although these lines have not been obtained in the laboratory.

III. STRUCTURE OF THE PLANETARY NEBULAE

Since the foregoing identifications determine the chief constituents of the nebulae, it now becomes possible to attempt a consideration of the mechanism of radiation and the distribution of ionization in the different parts of the planetary nebulae.

As a first approximation, consider that a star whose surface temperature is 150,000° C. is surrounded by oxygen of very low density. As Zanstra[1] has pointed out, most of the absorption of radiation in a

[1] *Astrophysical Journal*, **65**, 1, 1927.

gas is photo-electric absorption of light whose frequency is greater than the ionization frequency of the atom or ion, each quantum absorbed completely ejecting an electron from the atom. Owing to the very great absorption coefficient of any atom or ion for this radiation of frequency greater than that necessary for ionization, no atom or ion can exist long in a region where such radiation is present in ap-

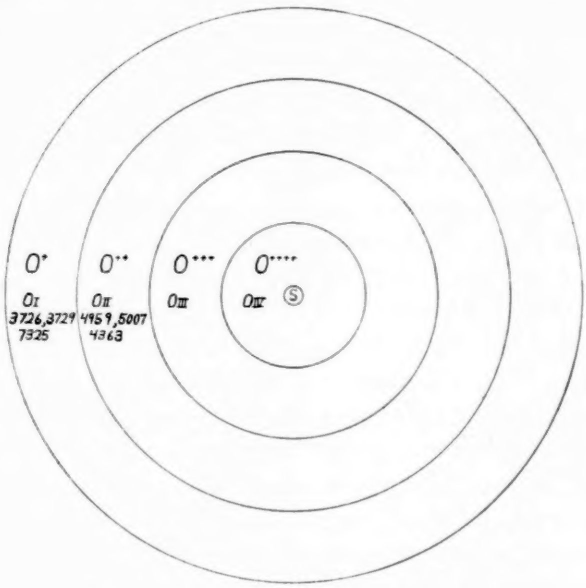

FIG. 2.—Cross-section of a planetary nebula

preciable strength. It is evident from Figure 1, which represents the distribution in frequency of the quanta emitted by a black body at 150,000° C., that the oxygen in the neighborhood of the star cannot exist in a stage of ionization lower than O^{++++}.

Occasionally, however, an electron unites with an O^{++++} ion and in so doing emits the O_{IV} spectrum. It is then almost immediately ejected again by the radiation below 160 A. This ejection of electrons made necessary by their continual return to the O^{++++} ion rapidly absorbs the light below 160 A while allowing longer wave-lengths to pass unaffected. After traversing a certain distance, represented by the radius of the first circle in Figure 2, the radiation of wave-length below 160 A is so completely absorbed that O^{+++} can exist in the re-

gion outside this first circle. The return of electrons to this ion causes the O_{III} spectrum to be emitted in this second region. This electron is immediately ejected by the light in the 160–225 A region, but in so doing causes the rapid absorption of the remaining light below 225 A. This absorption becomes complete by the time the second circle is reached, thus enabling the O^{++} ion to exist in the third region. Returns of electrons to these ions give rise to the O_{II} spectrum, but in turn causes the light in the 225–353 A band to be completely absorbed by the time the third circle is reached. In a similar manner, the O^{+} ion can exist in the next region, and the return of an electron to it causes the O_{I} spectrum to be emitted but completes the absorption of all of the starlight up to 911 A.

Thus it is seen that this mechanism gives a series of concentric shells whose diameter increases with the decrease of the ionization potential of the ions present in them. This gives a qualitative explanation of the differences in the size of the monochromatic images of a planetary nebula as observed with a slitless spectrograph.

This simple theory would seem to indicate that the lines due to each stage of ionization receive the energy of the band between its ionization frequency and that of the next stage, i.e., O_{I} gets the energy from the 911–353 A range, O_{II} from 353–225 A, O_{III} from 225–160 A, O_{IV} from 160–115 A, etc. If this is correct, one can then follow Zanstra[1] and assume that the number of quanta emitted in the 3300–5100 A range for a given stage of ionization are approximately equal to those absorbed in its appropriate band in the ultra-violet. From the ratio of the intensities of these stages it should be possible to calculate the temperature of the exciting star.

Of course, in any actual nebula ions other than those of oxygen are present, but these can be grouped with the oxygen ion having an ionization potential similar to their own. Thus H_{I} can be classed with O_{I} since it has almost identically the same ionization potential, and He_{II} can be classed with O_{III}, etc.

Hydrogen exhibits somewhat different characteristics from other atoms, since it has only a single electron and consequently can be only singly ionized. Thus, whenever an electron returns it always emits the H_{I} spectrum and then absorbs light up to 911 A even

[1] *Ibid.*

though the hydrogen atom is in the circle nearest the star. Owing to the great increase in the absorption coefficient in the immediate neighborhood of the absorption edge, however, most of this absorption takes place from the region immediately below 911 A rather than in the region normally absorbed by the more highly ionized oxygen ions. Nevertheless, this may result in the complete absorption of the energy in the 353–911 A range before the outer region is reached, and therefore may account for the non-appearance of the O_{I} spectrum to be expected in that circle. It also accounts for the rather small monochromatic H-images of the nebulae.

In Table III there are given the number of quanta (expressed as

TABLE III

RELATIVE NUMBER OF QUANTA AVAILABLE FOR EXCITATION OF SPECTRA

(Number in Percentage)

T	O_{I}	O_{II}	O_{III}	O_{IV}
1,000,000	2.5	3.7	5.3	8.8
500,000	8.2	11.4	13.9	17.5
300,000	18.9	20.4	19.7	18.1
200,000	31.9	26.0	17.6	10.4
150,000	42.2	25.0	11.9	4.4
100,000	51.0	15.3	3.5	.5
70,000	46.9	5.6	.5	....
50,000	32.2	1.0	....	....

percentage of the total emission) that are emitted between the various ionization frequencies of oxygen by stars at various temperatures. These then represent the relative number of quanta available for the production of the spectra of the various stages of ionization listed at the head of each column. It is evident that even the highest temperature listed (1,000,000° C.) does not permit enough quanta to be taken up by O_{III} and He_{II} to account for the great intensity of their lines relative to those of hydrogen as given in Table II. This discrepancy is made even more serious if the intensities are corrected for the fact that the 5007 and 4959 lines of O_{III} fall in a region that is only about one-half as active photographically as that where the hydrogen lines are located.

One possible cause of the abnormal strength of 5007 and 4959 and the other lines due to jumps from metastable states is that these are,

in most cases, the true resonance lines of these ions and require only 1.8–5.3 volts for their excitation (see Table I). Thus an O^{++} ion can be excited to emit the 5007 and 4959 line by an electron with a velocity of only 2.5 volts. That many electrons capable of causing excitation in this way are present in the nebulae is to be expected from the fact that the band of frequencies corresponding to the difference between two successive stages of ionization has a width corresponding to about 20 volts, since in oxygen the successive ionization potentials are 13.6, 35.0, 54.8, 77.0, 107.7, etc., volts. This means that the electrons ejected photo-electrically from the ions will have an excess velocity of from 0 to 20 volts, which must, in general, be used up in exciting resonance radiation in other atoms before the electron is slowed up enough to attach itself to an ion.

This mechanism causes these resonance lines of O_{III} to be omitted when an O^{++} ion is hit by an electron and the O_{II} lines when an O^{+} ion is hit. Consequently, the resonance lines of any particular stage of ionization are emitted in the next region farther out from the star than the one where that particular stage of ionization is emitted by a returning electron. Thus the 5007, 4959 lines of O_{III} occur in the region where the regular series lines of O_{II} are emitted, since this O_{III} pair is emitted by the excitation of the O^{++} ion by a 2.5-volt electron, while the regular O_{II} lines are produced by the return of an electron to this same ion. This, of course, is the next region larger than the one where the regular O_{III} lines are produced.

These considerations explain the relative sizes of the various monochromatic images found in the planetaries. Thus 3426.2 and 3346 of N_{IV} and O_{IV} have the smallest. Next comes 4686 of He_{II} and the 3313, 3342, 3444 lines of O_{III}. These are followed by He_{I}, 5007 and 4959, with the 3726 of O_{II}, the largest of all. The hydrogen lines vary somewhat from nebula to nebula, as would be expected from the differences mentioned above, but in general they are just smaller than 5007 and 4959.

The H-lines should also be stronger than would be expected from the absorption of the energy in the range 353–911 A, since every time an electron returns to O^{++}, O^{+++}, or O^{++++} at least one line is emitted in the region below 911 A which may be absorbed by a hydrogen atom. This enables the hydrogen lines to be excited indirectly

by light leaving the star in the range below 353 A even after this light has been effective in the excitation of the appropriate O_{II}, O_{III}, or O_{IV} spectrum.

All of these secondary effects make it very difficult to perform a quantitative calculation as to the temperature of the exciting star. However, the great intensity of the He_{II} and the N_{IV} and O_{IV} lines which cannot be enhanced by any of these secondary effects require so much radiation in the range below 225 A that a temperature of at least 100,000° C must be postulated for the hottest nuclei.

The mechanisms suggested above show that in general no radiation having a frequency greater than the ionization frequency of hydrogen or a wave-length less than 911 A can leave the nebulae. All of the quanta of wave-length less than 911 A must be split up into quanta whose wave-lengths are greater than 911 A. Since a large part of the energy in the star's radiation is in the region around 250 A, this necessitates a manifold increase in the number of quanta given out by the nebula over the number received by it from the star. This helps to account for the very great brilliance of the planetaries relative to that of the central exciting star.

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WAVE-LENGTHS IN THE ARC SPECTRUM OF CHROMIUM IN INTERNATIONAL UNITS[1]

By DONALD FOSTER

ABSTRACT

The wave-lengths of 277 chromium lines, in the region to the violet of λ 4352, are given. The errors in the wave-lengths are not larger than a few thousandths.

A convenient source is described.

I. INTRODUCTION

A complete account of investigations on the spectrum of chromium which were published previous to 1919 is given by Kayser in Volume **5** of his *Handbuch der Spectroscopie.* The measurements collected there were made before the importance of definitely adjusted and controlled conditions in the arc was recognized. The data of Stüting for the red are based on the standards of Fabry and Buisson. The other values are based on Rowland's system. More recently, wave-lengths in the ultra-violet, beyond the range of the present International Secondary Standards, have been given by S. Piña de Rubies[2] and by A. B. McLay.[3]

While it is possible partially to correct the errors due to the older standards, the corrected values are in doubt to the extent of two- or three-hundredths of an angstrom. The errors due to large currents and to pole effect in the arc are of unknown magnitude.

For these reasons it has seemed desirable to supplement the data on this element by redetermining the wave-lengths, using the new interferometer standards, and a source which complies with the specifications of the International Astronomical Union.

This has been done by J. Hall in his Bonn dissertation (1921), but his results were never published in any journal. An abstract containing only the measured wave-lengths was made available in 1924, shortly after the present measurements were completed, by

[1] A dissertation presented for the degree of Doctor of Philosophy in Yale University.

[2] "Nueva rayas del cromo en el espectro de arco en el aire entre 2300 y 1890 U.A.," *Anales de la Sociedad Española de Fisica y Quimica,* **15**, 110, 1917.

[3] "Ultra Violet Spectrum of Chromium," *Transactions of the Royal Society of Canada,* **17**, Sec. 3, 137–139, 1923.

the publishing of Volume 7 of Kayser's *Handbuch*. The author is indebted to Professor Kayser for a letter which supplies the information that Hall used a carbon arc with introduced chromium salts. The arc was operated with a length of 15–20 mm, 150 volts, and 3–5 amp. Only the middle part of the arc was used. The spectrograms were taken as far as possible in the second order of a large grating made by Rowland. The errors were thought to be not larger than a few thousandths angstroms except for the longer wave-lengths where the standards are poor. In the new edition of his *Hauptlinien*, however, Kayser has given many of these wave-lengths only as far as the second decimal place.

II. APPARATUS AND METHODS

The concave grating used was ruled by Anderson. It has a radius of curvature of about 640 cm (21 ft.) with 590 lines per millimeter (15,000 per inch). The linear dispersion in the second order is about 1.32 A per millimeter. The grating is mounted according to Rowland's plan, and the whole instrument is characterized by great rigidity. The grating-room is so designed and situated as to be practically free from changes in temperature.

Seed 26 plates 25.4 cm long by 3.2 cm wide were employed in photographing the spectra. A hydrochinone developer, described by L. E. Jewell,[1] was used.

The occulter, by means of which three spectrograms could be taken on a single plate, was mounted independently of the camera. It was operated from the arcroom by means of a string. The plates showed no evidence of mechanical shift.

In the arc, copper rods 7 mm in diameter were used as electrodes. A polished hemispherical cup was turned in the end of the lower rod, which was invariably the positive pole. Bits of iron and chromium were placed in the cup, and fused by means of the arc until a bead of the mixed metals filled the cup. The proportion of chromium to iron in the bead was so adjusted that few lines of the iron spectrum weaker than the standards appeared on the plates. The upper electrode was turned to a long rounded point which became plated with iron and chromium from the bead. As a result, no lines of the

[1] *Astrophysical Journal*, 11, 240, 1900.

copper spectrum were present in the radiation. With copper rods of this size, the heat of the arc is conducted away so rapidly that no cooling device is necessary.

Both the iron and the chromium were electrolytically prepared. The iron was very kindly supplied by the Bureau of Standards.

The normal conditions recommended by the Committee on Standard Wave-Lengths of the International Astronomical Union[1] were rigorously fulfilled. The current in the arc was kept at about 4.5 amp, and the length of the arc was 12–15 mm. The length could be accurately observed by the aid of a large image of the arc projected on the wall opposite the slit. In order to select the central zone of the arc, two diaphragms were used—one at the arc, the other over the quartz lens which focused the light on the slit. The diaphragm placed near the arc was made of lavite with a rectangular slot of 1 mm vertical height. This material is entirely satisfactory for the purpose, as it enables the diaphragm to be placed so near the arc that the height of the slot may be made nearly as great as the segment of the arc to be selected. Incidentally, the spectra for the regions near the positive and negative poles have been photographed on the same plates with those for the middle of the arc; but the method is not thought suitable for study of the pole effect.

A doubly enlarged image of the source was projected on the slit by means of a quartz lens. The vertical aperture of the lens was limited by a diaphragm to 1 cm. Under these circumstances the light which reached the slit came from a central zone not over 1.25 mm in length. The width of the slit was 0.017 mm, which corresponds to an average slit factor[2] of 2.

The average time of exposure for light from the center of the arc was about eight minutes. Some short exposures were made, with the percentage of iron in the arc considerably increased, in order to obtain images of the stronger lines of proper photographic density for accurate measurement.

The region λ 4352–λ 3358 was photographed in the second order. Lines in the overlapping third order, λ 2894–λ 2663, obtained on the

[1] *Transactions of the International Astronomical Union*, **1**, 36, 1922.

[2] Schuster, *Astrophysical Journal*, **21**, 197, 1905.

same plates, were compared with the standard lines belonging to the less refrangible region.

The plates were measured on a Gaertner comparator which has been modified and tested by several skilled observers. The errors in the screw are much smaller than the attainable degree of accuracy in measuring the spectrograms. Each plate was measured twice in one direction and then reversed and measured twice in the other direction.

Ordinarily seven secondary standards were included in a group of observations. Correction curves were plotted from the deviations from accepted values of the interpolated wave-lengths for the intermediate standards. The corrections, though negligible for λ 4352–λ 4076, amounted to an average of 0.0015 A for λ 4076–λ 3358, with a maximum value of 0.005 A.

The probable errors of the comparator measurements are less than 0.001 A in almost all cases. However, it cannot be said that the wave-lengths obtained are accurate to this extent. The small number of standards available in the lengths of spectra employed makes the use of correction curves somewhat difficult and uncertain; but it is indicated that the greatest source of uncertainty may be in the secondary standards themselves. This has already been pointed out by W. F. Meggers, C. C. Kiess, and Keivin Burns[1] who found for twenty-three lines belonging to groups *d* and *c* the secondary standards *minus* their new values averaged +0.0072 A. For forty-four lines of groups *a* and *b* the average difference between the international values and their redetermination was +0.0029 A.

The method of coincidences employed for the region of shorter wave-lengths calls for corrections to the wave-lengths obtained by interpolation, which are unnecessary when the unknown line and the standard belong to the same order. When the spectrum is photographed in air at other than normal density, the deviation of the index of refraction of the air from its normal value is a function of the wave-length. The necessary corrections were obtained from the tables of the Bureau of Standards, as published in their *Bulletin 327*. These tables are constructed for the correction of wave-length

[1] "Redetermination of Secondary Standards of Wave-Length from the New International Iron Arc," *Scientific Papers of the Bureau of Standards*, No. 478, 1924.

TABLE I

Present Work		Int.	Hall	Clodius	Present Work		Int.	Hall	Clodius
4351...	765	10	770	793	4203...	593	4*s*	587	596
4351...	059	8	058	050	4200...	106	1	097	116
4346...	829	5*s*	832	835	4198...	526	4	522	536
4345...	131	2	103	084	4197...	236	3	232	243
4344...	502	10	510	513	4194...	952	3*s*	948	948
4343...	176	2*s*	176	195	4193...	667	4*s*	662	668
4340...	131	4	134	147	4192...	107	1	102	106
4339...	880	1*u**	....	....	4191...	751	1	746	....
4339...	719	1*u*	718	732	4191...	275	5	267	276
4339...	450	10	452	460	4190...	131	1	126	145
4338...	780	1	801	780	4184...	901	2	895	....
4337...	558	10*s*	565	573	4179...	262	5*v*	251	256
4325...	068	3	074	080	4177...	953	1	....	....
4323...	522	1	524	523	4174...	806	5	801	816
4319...	637	1	651	649	4172...	763	1*U*	773	....
4305...	448	1	465	458	4170...	195	1	204	202
4301...	181	3	187	180	4169...	831	1	838	843
4300...	508	3	514	500	4165...	517	3	524	519
4299...	713	3	728	722	4163...	627	5*v*	625	625
4297...	741	3	746	743	4161...	419	3*s*	421	425
4297...	053	2*s*	058	056	4153...	822	5*v*	819	814
4295...	754	5	762	752	4131...	360	1	356	370
4293...	553	1*u*	573	590	4129...	253	4*U*	368	....
4291...	965	1	970	962	4127...	639	3	640	....
4289...	724	20*R*	726	736	4127...	287	1	298	307
4288...	370	3	388	395	4126...	918	1	923	925
4284...	722	1	737	738	4126...	519	7*s*	518	519
4280..	412	5*s*	411	404	4123...	386	3	391	391
4274...	803	20*R*	802	808	4122...	163	1	165	178
4272...	911	3*u*	921	927	4121...	821	2	821	....
4271...	070	2	062	....	4120...	617	2	620	622
4269...	959	1	950	....	4109...	583	4*s*	583	577
4263...	146	5	144	139	4104...	864	3	865	872
4262...	127	1	126	133	4077...	673	1*s*	676	696
4261...	624	1	618	607	4077...	085	3*s*	087	094
4261...	349	4	366	352	4076...	054	1	059	065
4255...	509	3*s*	502	496	4066...	945	4*s*	940	....
4254...	342	20*R*	341	341	4065...	716	1	716	707
4252...	233	1*s*	245	233	4058...	784	5	781	763
4240...	712	4*s*	706	706	4052...	762	1*h**	....	....
4238...	958	3	961	....	4048...	790	6*s*	790	773
4230...	475	1	483	477	4043...	123	1*h**	....	....
4226...	756	4*s**	....	....	4039...	099	5*s*	099	095
4222...	729	2*u*	740	730	4035...	031	2*u**	....	....
4221...	574	2*s*	564	573	4030...	679	1	684	....
4217...	614	2*v*	626	....	4030...	637	1	....	....
4216...	364	2*u*	367	355	4027...	105	4*s*	105	100
4213...	176	1	171	165	4026...	172	4*s*	175	171
4211...	349	1*u*	351	355	4025...	010	3	012	007
4209...	758	4*s*	753	761	4022...	269	1	266	258
4209...	369	5	364	369	4018...	198	2	216	....
4208...	358	2*s*	352	360	4012...	472	2	477	478
4204...	467	2*s*	464	470	4003...	065	1*	....	....

* Unidentified.

TABLE I—*Continued*

Present Work		Int.	Hall	Clodius	Present Work		Int.	Hall	Clodius
4001...	443	4*s*	466	439	3834...	728	1	748	730
3999...	025	2*s*	....	....	3831...	032	2	030	004
3993...	963	1	972	968	3825...	387	2*u*	408	389
3992...	845	5	851	839	3823...	520	2*s*	521	503
3991...	674	4*s*	677	667	3819...	570	4	578	563
3991...	122	8	124	129	3818...	485	2	480	464
3989...	983	3	988	984	3817...	849	1	848	808
3984...	337	4	342	337	3815...	439	3	439	424
3981...	240	2	241	236	3812...	255	1	258	241
3979...	793	1	798	800	3807...	923	1	925	918
3978...	681	2	686	677	3804...	802	5*s*	802	788
3976...	663	9	667	675	3797...	721	2	711	706
3971...	257	3	262	....	3797...	132	1	127	125
3969...	752	10	750	749	3794...	615	1	608	603
3969...	069	3	065	060	3793...	875	2	875	869
3963...	693	10	692	692	3793...	291	1	289	284
3953...	158	1	166	164	3792...	145	2	136	138
3952...	398	1	398	402	3791...	381	2	378	361
3946...	991	4	....	....	3790...	453	1	451	454
3941...	498	8*v*	494	490	3768...	738	3*s*	733	731
3928...	646	8	641	639	3768...	246	5	241	238
3927...	564	1*u*†	....	....	3758...	052	2	047	025
3925...	648	1	....	....	3757...	667	5*s*	661	656
3921...	024	7	023	036	3757...	172	1	170	176
3919...	167	10*v*	160	169	3748...	991	5	49 001	595
3917...	600	1	603	594	3748...	609	2	604	....
3916...	243	5*s*	238	235	3744...	495	2*u*	490	484
3915...	846	3	857	847	3743...	880	7	878	....
3908...	762	8	757	753	3743...	582	4*v*	561	583
3903...	162	3	163	159	3732...	034	4	029	027
3894...	038	6	040	043	3730...	808	4	802	796
3891...	936	1	93–	920	3712...	952	1	944	....
3886...	801	7	797	780	3688...	454	1	450	456
3885...	218	7	218	206	3686...	797	3*U*	836	839
3883...	640	1	660	645	3685...	544	2*u*	570	567
3883...	292	7	289	....	3666...	640	4*s*	646	642
3881...	211	1	246	218	3665...	982	2	979	....
3879...	209	1*u*	245	218	3663...	206	6	216	217
3878...	814	1*u**	....	....	3663...	196	2	....	....
3876...	802	1*u**‡	....	....	3656...	270	5*v*	265	264
3874...	520	2*U*	570	....	3653...	913	6	930	927
3863...	974	1*h**§	....	....	3648...	999	5	49 017	49 017
3857...	627	5	641	613	3648...	535	1	546	548
3855...	573	4	582	561	3646...	161	1	161	163
3855...	289	3	293	....	3641...	834	6	844	843
3854...	223	6	232	202	3641...	471	3*s*	479	490
3852...	217	5*s*	214	192	3639...	804	7	798	821
3849...	535	4*u*	535	510	3636...	594	6*v*	588	598
3849...	371	5*u*	35–	340	3635...	277	1	278	248
3848...	982	4	996	969	3632...	849	4*v*	841	849
3841...	278	5*u*	295	271	3619...	467	1	461	456
3836...	072	1*h*	087	071	3614...	339	3	....	....

† *Au?* § One observation only. ‡ *Os.?*

TABLE I—*Continued*

Present Work		Int.	Hall	Clodius	Present Work		Int.	Hall	Clodius
3609...	485	2	482	475	3453...	329	5*s*	328	327
3608...	405	1	401	433	3447...	757	1*h*	761	767
3605...	331	10*R*	330	337	3447...	431	5*s*	428	436
3603...	748	3*u*	739	786	3445...	617	5*v*	604	609
3601...	665	6*s*	654	661	3441...	437	3*u*	446	444
3593...	489	10*R*	483	488	3436...	192	4	185	193
3578...	689	10*R*	688	690	3433...	601	7	596	599
3574...	938	3	940	....	3433...	313	4*s*	307	312
3574...	805	4	805	791	3421...	212	4*s*	210	220
3574...	037	3*u*	039	046	3408...	770	6*s*	766	769
3572...	746	1	743	753	3403...	323	4*s*	319	322
3566...	164	4*UU*	66	164	3382...	684	1	678	676
3481...	535	1	531	528	3368...	056	5	049	041
3481...	305	1	299	304	3360...	300	1	295	296
3467...	716	1	718	720	3358...	509	3*s*	503	499
3455...	602	3*s*	607	603					

TABLE II

Present Work		Int.	Hall	Clodius	Present Work		Int.	Hall	Clodius
2894...	172	2*s*	178	168	2780...	695	1*UR*	702	687
2889...	279	2	267	262	2769...	904	2*U*	915	906
2889...	245	2	....	....	2766...	536	3	542	537
2886...	995	2*s*	87 001	890	2764...	360	1*h*	364	....
2879...	271	1	279	273	2757...	721	1	728	711
2877...	979	1	983	980	2757...	091	1*h*	100	092
2876...	245	1	250	247	2751...	866	2	873	860
2875...	991	2	998	992	2750...	726	2	731	725
2873...	483	1*s*	493	482	2748...	983	2	993	979
2867...	647	3*s*	655	641	2743...	640	1*h*	647	645
2866...	744	4*s*	748	746	2731...	909	3*U*	914	900
2865...	107	3*s*	112	111	2726...	512	1*r*	524	495
2862...	571	2*u*	575	563	2698...	690	1	695	680
2860...	931	2	940	936	2698...	410	1*u*	417	403
2855...	675	4	682	672	2691...	044	1	050	031
2843...	248	2*r*	254	237	2677...	160	6	171	147
2835...	650	1	640	643	2671...	811	1*h*	818	792
2835...	619	1	....	....	2663...	421	1	430	404

measurements in terms of the primary standard in air at 15° C. and 760 mm. The difference between the correction for the unknown line and that for the secondary standard is the quantity to be added to the computed wave-length.

To eliminate impurities, each line was compared with the chromium tables in Kayser's *Handbuch*. Lines suspected as due to possible impurities were compared with Kayser's *Tabelle der Haupt-*

linien. Scarcely any lines due to impurities were present. It is possible that a few of the lines in the tables, not observed by others, may be due to Lyman ghosts.

The results are given in terms of the secondary interferometer standards published in the report of the International Astronomical Union for 1922.

III. RESULTS

In the tables the abbreviations, following the estimated intensities, have the following signification:

u Diffuse	*v* Shaded toward the violet
U Very diffuse	*r* Narrow reversal
h Hazy	*R* Broad reversal
l Shaded toward the red	*s* Sharp

The values of Clodius given for comparison have been expressed in international units by means of the usual corrections.

I wish to express my thanks to Professor H. S. Uhler for many helpful suggestions.

SYSTEMATIC DEVIATIONS FROM THE MEAN STELLAR DISTRIBUTION[1]

By FREDERICK H. SEARES AND MARY C. JOYNER

ABSTRACT

Deviations from the symmetrical mean distribution.—Values of O−C in log N_m for m=9.0, 11.0, 13.5, 16.0, and 18.0 have been calculated for 10° intervals in galactic longitude in latitudes 0°, 5°, 10°, 20° . . . . 70°, north and south (Figs. 9–13, Tables XV*a–e*). The deviations thus found refer to the mean distribution in Table XVII, *Contribution* No. 301, here reprinted as Table XIV. The data are from the *Mount Wilson Catalogue of the Selected Areas*, from numerous zones of the *Astrographic Catalogue*, and from the *Harvard-Groningen Durchmusterung*. The counts from the *Durchmusterung* were used only for $\delta = -30°$ to $-90°$. They refer to limiting magnitude 16.86, assumed to be constant for all the fields. This assumption, unavoidable at present, introduces considerable uncertainty; but a similar treatment of the counts for $\delta = -15°$ to $+90°$ shows that the fluctuations in the limiting magnitude are not large enough to mask the characteristic features of the distribution.

Interpretation of the deviations.—The principal irregularity is of the form

$$\Delta = a + b \cos (\lambda - L') .$$

The values of the parameters (Table XVI, Figs. 14–16) for different zones of latitude are such as would be expected from a *displacement of the sun* from the center of an *approximately symmetrical stellar system.* The asymmetry in latitude, $\frac{1}{2}(a_N - a_S)$ (Table XVII, Fig. 14), depends on the limiting magnitude; for m=15 and fainter, it is negligible, whence it follows that the *solar system is almost exactly in the galactic plane defined by the faint stars.* The longitude of the center of the stellar system, L', also depends on the limiting magnitude (Fig. 16). Systematic differences in b and L' for northern and southern latitudes indicate the necessity of a *correction to the adopted position of the galactic pole* (Gould). The discussion is continued in *Contribution* No. 347.

The mean distribution of stars derived in *Contribution* No. 301[2] neglects differences between northern and southern galactic hemispheres and systematic deviations in longitude. Although sufficiently accurate for many purposes, the results thus found are only a first approximation, based on the assumption that the sun is at the center of a stellar system having spheroidal symmetry. To obtain a further approximation we may study the deviations of the density observed in different parts of the sky from the mean symmetrical distribution.

Various investigators have already dealt with different aspects of this question. For example, Nort,[3] Plummer,[4] and Pannekoek,[5]

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 346.

[2] Seares, van Rhijn, Joyner, and Richmond, *Astrophysical Journal*, **62**, 320, 1925.

[3] *Recherches astronomiques de l'Observatoire d'Utrecht*, **7**, 1917.

[4] *Monthly Notices, R.A.S.*, **78**, 668, 1918.

[5] *Ibid.*, **79**, 333, 1919.

by means of the Harvard "Map of the Sky," have shown the existence of periodic irregularities in the distribution of stars brighter than the eleventh magnitude; and, more recently, Pannekoek[1] has discussed the peculiarities of distribution revealed by the *Durchmusterung* catalogues. Kreiken,[2] with the aid of the *Harvard-Groningen Durchmusterung of the Selected Areas* and the *Draper Catalogue*, has studied the variation of density with longitude for stars near the Milky Way; while Turner[3] has analyzed the results of numerous counts in zones of the *Astrographic Catalogue*. Somewhat more restricted in their scope are studies of special regions, notably those of the great obscured area in Taurus, by Dyson and Melotte,[4] and of the star cloud in Cygnus by Pannekoek.[5]

The data from the *Mount Wilson Catalogue* used in *Contribution* No. 301 are also suited to the investigation of systematic irregularities and have the advantage that they include counts of stars on a reliable scale of brightness to a low limiting magnitude. These counts, however, do not extend south of declination $-15°$; and the fields, being small and in the Selected Areas, are considerably affected by local irregularities and too widely spaced for a satisfactory determination of the rapid changes in density occurring in certain parts of the sky. Nevertheless, these data are fundamental and contribute much to our knowledge of stellar distribution.

The Astrographic Zones for which detailed counts are now available cover the sky more satisfactorily, but still leave important gaps, as shown by Table VIII,[6] and include no stars fainter than about 13.5. Moreover, since the individual counts extend over a full hour

[1] *Researches on the Structure of the Universe*, Amsterdam, 1924.

[2] *Monthly Notices, R.A.S.*, **85**, 499, 985, 1925. In *ibid.*, **86**, 665, 1926, Kreiken reports a further investigation based on the Mount Wilson counts in *Contribution* No. 301 and the *Harvard-Groningen Durchmusterung*.

[3] *Monthly Notices, R.A.S.*, **85**, 610, 1925.

[4] *Ibid.*, **80**, 3, 1919.

[5] *Proceedings of the Amsterdam Academy of Sciences*, **13**, 239, 1910; *Monthly Notices, R.A.S.*, **79**, 500, 1919. See also Easton, *Bulletin of the Astronomical Institutes of the Netherlands*, **1**, 157 (No. 27, 1922).

[6] The Oxford zones, $+31°$, $+30°$, $+29°$, $+27°$, $+26°$, and $+25°$, have appeared since *Contribution* No. 301 was prepared for publication. A number of other zones for which no detailed counts, but totals only, are available, can also be used for the discussion.

in right ascension, rapid changes in density are smoothed out to an undesirable degree. In spite of these limitations, the Astrographic Zones clearly indicate the characteristic deviations from the mean distribution. Turner[1] has examined these data in detail, but a rediscussion is desirable, for the galactic concentration has in the meantime been redetermined and the counts themselves have been reduced to the international scale.[2]

To extend the investigation to faint stars in declinations south of $-15°$, we may use provisional results from the *Harvard-Groningen Durchmusterung*, reduced to the international scale. The limiting magnitude, 16.86, is considerably brighter than that of the *Mount Wilson Catalogue*, but at present no other data are available.

THE SELECTED AREAS

a) *Selected Areas 1–139.*—The *Mount Wilson Catalogue* gives the magnitudes on the international photographic scale for stars in Areas 1–139. Counts of these stars, and of about thirty-eight hundred faint stars omitted from the *Catalogue* because they appear on only a single plate, are given in Tables I*a* and I*b*, *Contribution* No. 301. These tables show the total numbers of stars brighter than $m=13.0$ and the numbers in each succeeding half-magnitude interval, down to the limit for the field in question. The values of log N_m for different limiting magnitudes have been found from these data for each area and compared with the mean distribution in Table XIV[3] below. Since the totals to $m=13.0$ are small, those for $m=14.0$ have also been compared with the mean distribution. For many of the areas the numbers are complete to $m=18.5$ or fainter, and in deriving the mean distribution the counts for the remaining fields were completed to this limit; but here it is safer not to use the data for stars fainter than $m=18.0$. On the other hand, it is unnecessary to make comparisons for every half-magnitude, or even for every magnitude. The limits used are $m=13.0$, 14.0, 16.0, 18.0.

Table I illustrates the method of combining the data. The values in the first two lines are the cumulative sums for S.A. 1, found from

[1] *Loc. cit.*

[2] *Mt. Wilson Contr.*, No. 305; *Astrophysical Journal*, **63**, 160, 1926.

[3] For convenience the mean distribution given in Table XVII of *Contribution* No. 301 is reprinted as Table XIV of the present paper.

Tables I*a* and I*b*, *Contribution* No. 301, completed to $m=18.0$ with the aid of Table V of the same *Contribution*. These totals refer to fields of 0.1154 and 0.0625 sq. deg., respectively. The corresponding numbers for an area of 1 sq. deg. are in the third and fourth lines of the table. These numbers are systematically too large, because each Selected Area was centered on a bright star.[1] The corrections given in the last column of Table V, *Contribution* No. 301, are nine stars for Mount Wilson and sixteen for Groningen. Thus for $m=13.0$ the corrected totals are $35-9=26$ and $64-16=48$, respectively, with a mean $N_m=37$. Hence, log $N_m=1.57$, whereas the mean distribution, Table XIV, gives 1.76. The difference O−C in log N_m is

TABLE I

DEVIATIONS FROM MEAN DISTRIBUTION FOR S.A. 1, GAL. LAT. +28°

	13.0	14.0	16.0	18.0
No. M.W.	4	5	38	176
No. Gr.	4	5	21	97
N_m, M.W.	35	43	329	1525
N_m, Gr.	64	80	336	1552
Mean N_m	37	49	320	1526
Obs. log N_m	1.57	1.69	2.51	3.18
Mean log N_m	1.76	2.12	2.78	3.35
O−C	−0.19	−0.43	−0.27	−0.17

therefore −0.19. Since the deviations for the other limits are also negative, the observed densities for S.A. 1 are systematically below the average for the galactic latitude in question.

The large percentage differences in the Mount Wilson and Groningen counts for $m=13.0$ and 14.0 arise from the small numbers actually counted. Differences of only one or two stars in the original numbers fully account for the discordances. For the fainter limits the agreement is generally excellent.

The results thus found for S.A. 1–139 are collected in Table II. For any given area, the four deviations usually have the same sign; if the observed density for any magnitude limit exceeds or falls short of that corresponding to the mean distribution, the same peculiarity generally appears in the densities for the other limits. Irregularities

[1] Cf. *Mt. Wilson Contr.*, No. 301, p. 18; *Astrophysical Journal*, **62**, 337, 1925.

TABLE II

DEVIATIONS IN LOG N_m, S.A. 1–139
(Unit = 0.01)

S.A.	α 1900	δ 1900	m 13.0	m 14.0	m 16.0	m 18.0
1	120°0	+89°0	−19	−43	−27	−17
2	1.2	75.3	−26	−35	−11	−12
3	62.0	75.0	−21	−11	−10	−13
4	121.0	74.8	+1	−6	+2	−8
5	187.0	75.0	+4	+13	−11	−6
6	243.5	74.8	−16	−27	−4	−5
7	306.0	75.2	−36	−33	−14	−8
8	15.0	60.2	−15	−11	−14	−17
9	46.0	60.3	−21	−30	−52	−50
10	77.0	60.2	−10	−24	−33	−27
11	106.8	60.0	−2	−5	−10	−15
12	135.8	59.7	−10	−23	−9	−22
13	165.8	59.8	−8	−21	−14	−11
14	200.5	59.5	−70	−20	−4	+4
15	229.2	59.8	+23	+2	+6	+6
16	262.2	59.8	−12	+1	+2	−1
17	290.8	60.2	0	−3	−6	−5
18	321.0	60.2	0	−3	−9	0
19	350.8	60.0	+6	+7	−13	−27
20	10.0	45.3	+14	+19	+4	−9
21	24.0	45.0	+16	+1	−1	−4
22	39.5	45.2	−8	−1	−4	−7
23	54.8	45.0	−18	−27	−30	−25
24	69.8	44.8	−24	−37	−26	−36
25	84.2	44.8	−47	−36	−18	−29
26	99.0	44.8	−14	+12	−6	−15
27	114.5	44.8	+17	−3	−6	−13
28	130.0	45.0	+30	+13	−8	−16
29	144.8	44.8	+21	+1	−5	−17
30	159.2	45.2	+15	−2	−6	−13
31	174.2	44.7	+19	+3	+2	0
32	192.8	44.8	+6	+8	−14	−4
33	207.5	45.2	+15	+13	−3	+11
34	222.0	45.0	+24	+4	+4	+6
35	237.2	44.8	−17	+21	+2	+17
36	251.5	45.3	−93	0	0	−6
37	267.2	45.0	+12	+20	+10	+7
38	281.5	45.2	+35	+29	+12	+9
39	296.8	44.8	+14	+3	+17	+27
40	311.8	45.0	−1	−7	−11	−6
41	327.5	45.0	+12	+8	+14	+19
42	342.2	45.2	0	−10	+1	+5
43	357.5	44.8	−11	+4	−1	−9
44	6.0	30.2	+25	+6	+2	−4
45	21.5	30.2	+12	+11	−1	−7
46	37.2	+30.2	−10	−15	−15	−24

TABLE II—*Continued*

S.A.	α 1900	δ 1900	*m*			
			13.0	14.0	16.0	18.0
47	50°8	+30°0	−185	−163	−102	− 80
48	65.8	30.2	− 30	− 33	− 25	− 27
49	81.0	29.7	− 15	− 11	− 19	− 10
50	96.0	29.8	+ 2	+ 2	0	− 5
51	111.0	30.0	− 22	+ 4	+ 3	− 6
52	126.5	30.0	− 21	− 39	− 19	− 18
53	141.2	30.0	− 78	− 39	− 14	− 13
54	156.0	30.0	− 18	− 8	− 14	− 25
55	172.5	30.0	+ 13	+ 6	− 10	− 15
56	179.5	29.7	+ 10	+ 16	+ 3	+ 4
57	196.0	30.0	− 39	+ 7	+ 11	− 4
58	210.0	29.7	+ 22	+ 9	+ 8	+ 9
59	225.5	29.8	− 20	− 3	+ 1	+ 16
60	239.0	29.8	+ 17	+ 5	+ 6	+ 6
61	254.8	30.0	+ 19	+ 19	+ 14	+ 7
62	268.8	30.0	+ 35	+ 25	+ 10	+ 21
63	285.0	30.0	+ 7	+ 20	+ 25	+ 19
64	299.5	30.0	+ 31	+ 32	+ 26	+ 30
65	314.8	30.2	+ 18	+ 8	+ 12	+ 14
66	329.5	30.2	+ 8	− 4	+ 13	+ 4
67	345.2	30.2	− 2	− 6	− 3	− 10
68	2.8	15.3	− 30	+ 22	+ 5	− 6
69	18.8	15.2	− 13	− 21	− 12	− 22
70	34.0	15.0	− 35	− 54	− 26	− 25
71	47.8	15.0	+ 14	− 13	− 24	− 27
72	62.5	15.2	.....	− 98	− 57	− 21
73	78.5	15.0	−140	− 58	− 26	− 21
74	93.8	15.2	− 23	− 23	− 31	− 33
75	108.5	15.2	+ 5	+ 14	+ 8	− 9
76	123.8	15.0	− 15	− 1	− 4	− 15
77	137.5	14.5	− 10	+ 12	− 1	− 13
78	153.2	15.2	+ 15	+ 6	− 15	− 12
79	169.2	14.8	+ 15	− 1	+ 4	− 2
80	183.0	15.0	− 11	0	− 6	+ 1
81	198.0	14.7	− 22	− 4	+ 6	+ 4
82	213.5	15.3	− 42	− 29	− 33	+ 11
83	227.2	14.8	+ 10	+ 13	+ 5	+ 10
84	243.0	15.0	+ 17	+ 18	+ 18	+ 28
85	257.5	15.0	+ 13	+ 6	− 10	− 15
86	272.8	15.0	− 7	+ 20	+ 8	+ 11
87	287.8	15.0	− 28	− 25	− 29	− 13
88	302.5	15.2	+ 19	+ 30	+ 28	+ 25
89	317.0	15.2	+ 15	+ 7	+ 2	+ 12
90	333.0	15.2	+ 16	+ 6	+ 2	− 4
91	348.2	15.0	− 9	− 34	− 12	− 6
92	12.5	0.2	+ 7	+ 13	+ 5	− 2
93	27.5	.3	+ 41	+ 21	+ 6	− 11
94	42.8	+ .2	+ 36	+ 14	− 11	− 19
95	57.5	.0	− 94	−110	− 30	− 38
96	72.0	.0	− 17	− 20	− 9	− 21
97	88.0	0.0	− 29	− 38	− 51	− 48

TABLE II—*Continued*

S.A.	α 1900	δ 1900	m			
			13.0	14.0	16.0	18.0
98	101°8	−0°2	+36	+15	+10	+17
99	117.5	−.3	0	+12	+17	+15
100	132.2	−.2	+32	+18	+12	+5
101	148.0	.0	−10	+7	−7	−18
102	162.5	−.3	+4	+9	+3	+5
103	177.5	.0	−3	−9	+1	−1
104	189.5	.0	+27	−3	+4	+9
105	203.2	−.2	+19	+12	+9	+12
106	219.2	.0	−7	−29	+12	+9
107	233.5	.0	+12	+16	−4	+11
108	248.0	−.2	−13	−22	+3	+21
109	265.0	−.2	−47	−12	−10	+2
110	279.2	.0	−30	−58	−96	−118
111	293.2	+.2	−49	−18	−1	+16
112	309.2	+.2	+1	+12	+21	+21
113	324.2	.0	+8	−5	+1	−2
114	339.2	+.2	−40	−1	+5	+14
115	354.5	+0.3	+18	+13	−4	−9
116	3.2	−14.8	+19	+4	−5	−18
117	18.0	−14.7	−137	+1	−9	0
118	33.8	−14.8	+9	−2	−9	+7
119	47.5	−14.8	−35	+21	−2	−13
120	63.0	−15.2	0	−8	−17	−5
121	78.8	−14.8	−71	−13	−14	−17
122	93.0	−15.2	−12	−10	−12	−7
123	108.8	−15.0	+37	+25	+20	+14
124	123.5	−15.2	+18	+24	+10	+4
125	138.0	−15.2	+31	+19	+8	+1
126	154.0	−15.0	+8	0	−7	−8
127	168.2	−15.3	+9	−5	−3	+4
128	183.5	−15.2	+17	+11	−4	+4
129	198.5	−15.0	−5	+5	+1	+2
130	212.0	−15.2	0	−4	−2	+9
131	227.8	−15.2	+14	0	−4	+11
132	243.0	−15.0	−15	−18	−26	−3
133	258.2	−15.2	−68	−39	−24	+10
134	272.5	−15.0	+36	+24	+9	+1
135	288.2	−15.0	+12	+13	+31	+59
136	302.5	−15.2	+11	+8	+19	+32
137	317.5	−14.8	+10	+7	+6	+14
138	332.8	−15.0	0	+14	+5	+6
139	348.5	−14.8	−16	−1	−5	−4

of distribution thus extend over a considerable interval in magnitude.

Table II also reveals irregularities affecting large areas of the sky. For example, an extended region near the North Pole is of abnormally low density, while another, covered by S.A. 60–66 and 83–

89, systematically exceeds the mean density. The existence of these large-scale irregularities emphasizes the importance of data on the distribution of stars beyond the southern limit of the *Mount Wilson Catalogue* at declination -15°. Preliminary values were obtained from the *Harvard-Groningen Durchmusterung.*[1]

b) *Selected Areas 140–206.*—The magnitude scale of the *Harvard-Groningen Durchmusterung* is approximately that of *Harvard Annals*, **71**, No. 3. *Contribution* No. 289, Table III,[2] gives the reductions to the international photographic scale for S.A. 1–139. For the remaining regions the scale errors are unknown, but, as in the case of the northern areas, they probably vary considerably from field to field. It must first be demonstrated, therefore, that useful results may be obtained from these data without knowing the corrections for individual areas. This may be done by calculating the deviations from the mean distribution corresponding to the *Durchmusterung* counts for S.A. 1–139 and comparing the results with those from the *Mount Wilson Catalogue* given in Table II. If reasonable agreement is found, we may assume that the southern areas of the *Durchmusterung* will give a useful approximation for the distribution south of declination -15°.

Since the complete reduction of the *Durchmusterung* is in the hands of Professor van Rhijn, the comparison made here is based simply on total numbers of stars for each area as listed in the *Durchmusterung* itself. To proceed, we assume that these totals correspond to a constant limiting magnitude. For this we adopt the mean found by comparing the totals for S.A. 1–139 with the distribution given by the *Mount Wilson Catalogue.* Two limiting magnitudes must, in fact, be determined—one for the Metcalf and one for the Bruce telescope, for which, with minor exceptions allowed for in the calculation, the exposure times were one and two hours, respectively.

Aside from exposure and instrumental differences, the limiting magnitudes for the individual areas are affected by differences in plates, development, atmospheric conditions, and zenith distance of the fields, and by the fact that the stellar density in the small fields of the *Mount Wilson Catalogue* will not always be representative of

[1] *Harvard Annals*, **102**, **103**.

[2] *Astrophysical Journal*, **61**, 313, 1925.

TABLE III

DEVIATIONS FROM *Durchmusterung*, S.A. 1–139

S.A.	Lat.	Long.	log N_m	m_1	Δm_1	Δm_2	Δ log N_m
1	+28°	91°	2.58	15.35	+0.87	+0.03	−0.28
2	+13	88	3.01	15.70	+ .31	+ .24	− .20
3	+18	102	2.80	15.37	+ .10	+ .78	− .29
4	+32	107	2.65	15.77	− .07	+ .55	− .14
5	+42	91	2.48	15.68	+ .40	+ .17	− .16
6	+36	75	2.58	15.75	+ .14	+ .36	− .15
7	+20	76	3.04	16.30	+ .42	− .47	+ .02
8	− 2	92	3.26	15.88	+0.39	− .02	− .13
9	+ 3	106	2.94	15.00	+1.44	− .19	− .45
10	+13	118	2.90	15.40	+0.92	− .07	− .31
11	+27	123	2.88	16.20	+ .32	− .27	− .02
12	+41	123	2.39	15.35	+ .31	+ .59	− .26
13	+53	111	2.37	15.75	+ .52	− .02	− .14
14	+57	81	2.33	15.75	+ .15	+ .35	− .14
15	+48	62	2.59	16.32	− .21	+ .14	+ .02
16	+33	56	2.56	15.54	− .07	+ .78	− .21
17	+19	59	2.97	16.01	+ .18	+ .06	− .08
18	+ 6	68	3.49	16.61	+ .25	− .61	+ .13
19	− 1	81	3.39	16.18	+ .36	− .29	− .03
20	−17	89	3.03	16.10	− .12	+ .27	− .05
21	−17	99	3.24	16.67	+ .03	− .45	+ .14
22	−13	111	3.35	16.64	+ .11	− .50	+ .14
23	− 7	120	3.19	15.82	+ .83	− .40	− .15
24	0	128	3.16	15.55	+ .72	− .02	− .25
25	+ 9	133	3.02	15.42	+ .50	+ .33	− .30
26	+18	138	2.97	15.91	+ .18	+ .16	− .12
27	+29	141	2.74	15.85	+ .19	+ .21	− .12
28	+39	143	2.53	15.74	+ .28	+ .23	− .15
29	+50	141	2.47	15.95	+ .18	+ .12	− .08
30	+60	135	2.46	16.35	+ .22	− .32	+ .03
31	+68	123	2.22	15.70	− .08	+ .63	− .14
32	+72	84	2.42	16.57	+ .54	− .86	+ .08
33	+67	57	2.43	16.50	+ .11	− .36	+ .07
34	+59	43	2.51	16.50	− .15	− .10	+ .07
35	+49	39	2.56	16.25	− .07	+ .07	.00
36	+39	38	2.65	16.15	.00	+ .10	− .03
37	+28	39	2.72	15.78	− .32	+ .79	− .15
38	+18	42	3.14	16.45	− .35	+ .15	+ .07
39	+ 9	47	3.52	16.87	− .47	− .15	+ .22
40	0	53	3.25	15.80	+ .31	+ .14	− .16
41	− 8	61	3.36	16.36	− .48	+ .37	+ .03
42	−13	70	3.25	16.35	− .03	− .07	+ .04
43	−17	80	3.02	15.92	+ .03	+ .30	− .11
44	−32	85	2.65	15.80	− .07	+ .52	− .14
45	−32	101	2.72	16.00	+ .03	+ .22	− .08
46	−27	117	2.70	15.68	+0.48	+0.09	− .18
47	−21	127	2.44	14.60	+2.95	−1.30	− .54
48	−12	136	3.01	15.62	+0.69	−0.06	− .23
49	− 2	145	3.31	16.00	+ .53	− .28	− .09
50	+10	151	3.41	16.63	.00	− .38	+ .14
51	+22	156	2.90	16.00	− .09	+ .34	− .08
52	+35	161	2.82	16.45	+ .66	− .86	+ .06
53	+47	164	2.55	16.15	+0.40	−0.30	−0.03

TABLE III—*Continued*

S.A.	Lat.	Long.	log N_m	m_1	Δm_1	Δm_2	Δ log N_m
54......	+60°	166°	2.31	15.79	+0.52	−0.06	−0.12
55......	+74	166	2.31	16.19	+ .40	− .34	− .02
56......	+80	163	2.32	16.33	− .11	+ .03	+ .02
57......	+84	30	2.36	16.56	− .42	+ .11	+ .08
58......	+72	12	2.37	16.40	− .31	+ .16	+ .04
59......	+59	13	2.56	16.70	− .04	− .41	+ .12
60......	+47	15	2.55	16.19	− .21	+ .27	− .02
61......	+34	19	2.73	16.15	− .48	+ .58	− .03
62......	+23	24	3.17	16.92	− .31	− .36	+ .21
63......	+10	29	3.28	16.23	− .70	+ .72	− .01
64......	− 1	35	3.64	16.90	− .71	+ .06	+ .23
65......	−12	43	3.16	16.02	− .33	+ .56	− .09
66......	−21	53	3.17	16.70	− .39	− .06	+ .15
67......	−28	65	2.74	15.85	+ .10	+ .30	− .12
68......	−47	79	2.42	15.68	− .18	+ .75	− .16
69......	−47	102	2.33	15.38	+ .43	+ .44	− .24
70......	−42	121	2.38	15.36	+ .93	− .05	− .25
71......	−34	135	2.55	15.54	+0.83	− .12	− .21
72......	−24	146	2.51	14.95	+1.78	− .48	− .42
73......	−11	156	2.92	15.38	+0.71	+ .16	− .31
74......	+ 1	163	3.16	15.55	+ .86	− .16	− .25
75......	+14	170	3.25	16.44	− .23	+ .04	+ .07
76......	+27	177	2.70	15.68	+ .13	+ .44	− .18
77......	+39	183	2.52	15.69	+ .04	+ .52	− .16
78......	+54	192	2.41	15.92	+ .56	− .23	− .09
79......	+67	208	2.35	16.15	− .15	+ .25	− .03
80......	+76	240	2.24	15.95	+ .23	+ .07	− .08
81......	+75	301	2.33	16.30	−0.15	+0.10	+ .01
82......	+65	336	2.39	16.28	+1.27	−1.30	+ .01
83......	+53	347	2.52	16.33	−0.19	+0.11	+ .02
84......	+40	357	2.63	16.13	− .62	+ .74	− .03
85......	+27	4	2.82	16.05	− .26	+ .46	− .06
86......	+13	10	3.27	16.58	− .23	− .10	+ .12
87......	0	17	3.32	16.00	+ .08	+ .17	− .09
88......	−12	24	3.51	17.05	− .78	− .02	+ .29
89......	−23	33	3.12	16.78	− .07	− .46	+ .17
90......	−34	45	2.84	16.54	− .07	−0.22	+ .08
91......	−43	61	2.85	17.03	+ .43	−1.21	+ .22
92......	−62	95	2.52	16.64	− .19	−0.20	+ .10
93......	−58	123	2.61	16.84	− .22	− .37	+ .16
94......	−48	143	2.60	16.41	+0.41	− .57	+ .04
95......	−37	157	2.61	15.93	+1.03	− .71	− .09
96......	−25	166	3.00	16.53	+0.29	− .57	+ .09
97......	−11	174	2.93	15.33	+1.42	− .50	− .33
98......	+ 1	181	3.58	16.72	−0.28	−0.19	+ .17
99*.....	+15	188	3.00	15.80	− .49	+1.55	− .35
100......	+28	196	2.84	16.18	− .39	+0.46	− .02
101*.....	+41	206	2.53	15.80	+ .46	+ .60	− .28
102*.....	+52	221	2.80	17.35	− .16	− .31	+ .12
103*.....	+60	244	2.70	17.30	.00	− .44	+ .11
104*.....	+63	268	2.67	17.25	− .16	− .23	+ .09
105*.....	+60	295	2.66	17.15	− .40	+ .11	+ .07
106*.....	+51	320	2.88	17.63	− .40	− .37	+ .19
107*.....	+40	334	2.79	16.73	+0.15	−0.02	−0.03

TABLE III—*Continued*

S.A.	Lat.	Long.	log N_m	m_1	Δm_1	Δm_2	Δ log N_m
108*.....	+28°	343°	3.32	17.90	−0.43	−0.61	+0.29
109*.....	+14	353	3.41	16.92	+0.12	− .18	+ .02
110*.....	+ 1	0	2.73	14.41	+3.15	− .70	− .83
111*.....	−11	6	3.59	17.21	−0.24	− .11	+ .12
112*.....	−25	15	3.46	18.20	− .72	− .62	+ .39
113*.....	−38	24	2.82	16.70	.00	+ .16	− .04
114*.....	−49	38	2.66	16.65	− .40	+ .61	− .05
115*.....	−58	60	2.53	16.53	+ .24	+ .09	− .08
116......	−76	63	2.53	17.13	+ .50	− .77	+ .06
117......	−75	118	2.48	16.92	+ .17	− .23	+ .01
118......	−65	154	2.42	16.38	+ .04	+ .44	− .12
119......	−53	168	2.51	16.30	+ .32	+0.24	− .14
120......	−40	177	2.40	15.32	+ .42	+1.12	− .40
121......	−26	184	2.93	16.35	+ .55	−0.04	− .15
122......	−13	190	3.29	16.45	+ .30	+ .11	− .14
123......	0	198	3.65	16.92	− .50	+ .44	+ .02
124......	+13	205	3.37	16.62	− .21	+ .45	− .08
125......	+24	214	2.98	16.40	− .14	+ .60	− .13
126......	+35	226	2.78	16.39	+ .30	+ .17	− .13
127......	+42	241	2.62	16.18	.00	+ .68	− .18
128......	+47	260	2.77	16.98	.00	− .12	+ .03
129......	+47	282	2.88	17.43	− .08	− .49	+ .12
130......	+42	299	2.83	17.00	− .15	+ .01	+ .04
131......	+34	315	3.16	17.75	− .15	− .74	+ .24
132......	+24	326	3.26	17.32	+ .48	− .94	+ .13
133......	+12	336	3.30	16.42	+ .21	+ .23	− .15
134......	0	343	3.77	17.26	−0.15	− .25	+ .14
135......	−14	350	3.65	17.56	−1.36	+ .66	+ .23
136......	−26	356	3.24	17.44	−0.90	+ .32	+ .16
137......	−40	3	3.15	18.08	− .37	− .85	+ .32
138......	−53	12	2.82	17.48	− .24	− .38	+ .15
139......	−66	29	2.59	17.08	+0.16	−0.38	+0.05

*Bruce telescope.

that in the larger fields of the *Durchmusterung*. Rather large variations in the limiting magnitude are therefore to be expected, especially since the results depend on a single photograph for each area; but it will now be shown that in S.A. 1–139 these differences are not so large as to mask the characteristic features of the distribution. The distribution for S.A. 140–206 will then be calculated on the assumption that similar conditions hold for these areas.

The *Durchmusterung* values of N_m were taken from the last column of Tables XLVIII, VI, and VI of *Harvard Annals*, **101, 102,** and **103,** respectively. The corresponding values of log N_m are in the fourth column of Tables III and IV below. The limiting magnitudes m_1 interpolated from Table XIV, with log N_m and the galactic

latitude as arguments, are in the fifth column of Table III. In the mean, the limits for the Metcalf and Bruce telescopes are near the sixteenth and seventeenth magnitudes, respectively. Individual values, however, are appreciably affected by the departure of the

TABLE IV

DEVIATIONS FROM *Durchmusterung*, S.A. 140–206

S.A.	Lat.	Long.	log N_m	Δ log N_m	S.A.	Lat.	Long.	log N_m	Δ log N_m
140....	−81°	341°	2.57	+0.12	174...	+12°	248°	3.55	+0.10
141....	−85	211	2.23	− .20	175...	+16	258	3.47	+ .13
142....	−72	192	2.40	− .09	176...	+18	270	3.45	+ .17
143....	−60	192	2.56	− .04	177...	+16	282	3.66	+ .32
144....	−46	196	2.64	− .11	178...	+12	292	3.82	+ .37
145....	−34	199	2.83	− .10	179...	+ 6	301	3.69	+ .12
146....	−22	204	3.02	− .15	180...	− 1	309	3.91	+ .28
147....	− 9	209	3.50	− .02	181...	−10	314	3.89	+ .39
148....	+ 1	215	3.77	+ .14	182...	−20	318	3.86	+ .64
149....	+13	224	3.43	− .02	183...	−31	321	3.33	+ .35
150....	+21	233	3.15	− .04	184...	−41	323	3.10	+ .28
151....	+27	245	3.10	+ .07	185...	−52	321	2.85	+ .17
152....	+32	266	2.98	+ .02	186...	−61	314	2.84	+ .24
153....	+32	280	3.00	+ .07	187...	−69	297	2.61	+ .10
154....	+28	295	3.27	+ .24	188...	−57	261	2.50	− .13
155....	+21	307	3.67	+ .48	189...	−47	241	2.61	− .13
156..	+12	317	3.75	+ .30	190...	−33	236	3.16	+ .22
157....	+ 1	325	3.89	+ .26	191...	−19	239	3.06	− .19
158....	−10	331	4.06	+ .54	192...	− 6	248	4.12	+ .55
159....	−22	336	3.79	+ .60	193...	+ 1	261	4.34	+ .71
160....	−34	341	3.31	+ .38	194...	+ 2	272	4.08	+ .47
161....	−48	344	2.92	+ .18	195...	− 2	286	3.89	+ .27
162....	−60	347	2.72	+ .12	196...	−12	298	4.04	+ .59
163....	−73	345	2.53	+ .04	197...	−26	303	3.41	+ .33
164....	−73	271	2.52	+ .03	198...	−40	303	3.13	+ .30
165....	−69	245	2.41	− .09	199...	−53	291	2.72	+ .06
166....	−61	226	2.55	− .04	200...	−42	272	3.19	+ .39
167....	−51	219	2.56	− .13	201...	−35	255	2.85	− .06
168....	−40	217	2.66	− .17	202...	−20	256	3.20	− .02
169....	−29	218	2.95	− .05	203...	−13	267	3.46	+ .04
170....	−20	222	3.21	− .01	204...	−17	282	3.81	+ .51
171....	−10	226	3.48	− .02	205...	−32	287	3.02	+ .06
172....	− 1	232	3.67	+ .04	206...	−28	270	2.72	−0.22
173....	+ 6	239	3.74	+0.17					

true densities from the adopted mean values. To correct the interpolated limiting magnitudes for this disturbance, we assume that the true distribution deviates from the adopted mean by the amounts found from the *Mount Wilson Catalogue* and shown in Table II. Thus for S.A. 1, $m = 16.0$, the correction to the value of log N_m appearing in the mean distribution table is -0.27. The first

differences in Table XIV show that an interval of 1 mag. in m corresponds to a change of 0.309 in $\log N_m$. The limit $m_1=15.35$ for this area is therefore to be corrected by $0.27/0.309=0.87$ mag.; and, since the density for S.A. 1 is less than the mean, the correction is to be added, thus giving 16.22 for the revised limit. The corrections to m_1 for the Bruce telescope, for which the limiting magnitude is about 17, were calculated from the mean of the deviations for $m=16.0$ and 18.0 given in Table II.

Tables V and VI exhibit the internal consistency of these results. The first of these tables gives the means of m_1, of the corrections Δm_1, and of the corrected limiting magnitudes m for groups of ten areas (nine in one group), and also the mean residuals v_1 and v, referred to the respective means m_1 and m. In general, the corrected limits agree much better than the original values; the means of v and v_1 for the entire series, without regard to sign, are roughly in the ratio of 2:3. This improvement, by itself, indicates that, notwithstanding uncertainties in the limiting magnitude, the *Durchmusterung* counts show the general features of the distribution found from the *Mount Wilson Catalogue*.

Table VI, which gives the residuals in the mean corrected limiting magnitudes for groups of ten areas arranged in order of galactic latitude, shows no pronounced dependence of magnitude on latitude. By themselves the negative signs in high latitudes might suggest that measures in regions of low density have been pushed to a slightly lower limit than elsewhere; but the rich fields in and near the Milky Way show no corresponding excess of positive residuals. With the exception of the last residual for the Metcalf telescope, the accordance seems to be all that can be expected from the data.

The adopted limiting magnitudes[1] are 16.25 for the Metcalf telescope and 16.86 for the Bruce telescope. The corresponding residuals in m for the individual areas are given in the fifth column of Table III. Their average, without regard to sign, is ± 0.37 mag. This is equivalent to ± 0.12 in $\log N_m$, which indicates the uncertainty in the densities found from the *Durchmusterung* on the assumption of a constant limiting magnitude.

[1] The difference of 0.01 mag. shown by the mean in Table V arises from neglected decimals.

The deviations from the mean distribution given by the *Durchmusterung* for S.A. 1–139 can now be found by interpolating log N_m

TABLE V

MEAN LIMITING MAGNITUDE

S.A.	m_1	v_1	Δm_1	m	v
Metcalf Telescope					
1–10	15.62	+0.45	+0.49	16.11	+0.15
11–20	15.98	+ .09	+ .17	16.15	+ .11
21–30	15.99	+ .08	+ .32	16.31	− .05
31–40	16.26	− .19	− .05	16.21	+ .05
41–50	15.90	+ .17	+ .41	16.31	− .05
51–60	16.28	− .21	+ .08	16.36	− .10
61–70	16.12	− .05	− .16	15.96	+ .30
71–80	15.72	+ .35	+ .48	16.20	+ .06
81–90	16.40	− .33	− .10	16.30	− .04
91–98, 100	16.40	−0.33	+0.28	16.68	−0.42
Means	16.07	0.00	+0.19	16.26	0.00
Av. dev.	....	±0.225	....	....	±0.133
Bruce Telescope					
99, 101–109	16.98	−0.12	−0.13	16.85	+0.01
110–119	16.64	+ .22	+ .31	16.95	− .09
120–129	16.50	+ .36	+ .06	16.56	+ .30
130–139	17.34	−0.48	−0.25	17.09	−0.23
Means	16.86	0.00	0.00	16.86	0.00
Av. dev.	....	±0.27	....	....	±0.16

TABLE VI

RESIDUALS IN LIMITING MAGNITUDE GROUPED ACCORDING TO LATITUDE

Lat.	v	Lat.	v
1°	−0.01	34°	+0.07
6	− .20	39	+ .25
12	+ .17	44	+ .04
14	+ .17	50	− .06
19	− .05	56	− .10
25	− .30	64	− .12
28	+0.27	76	−0.21

from Table XIV with the mean limiting magnitude and comparing the results with the observed values in the fourth column of Table III. The differences, observed *minus* mean, are in the last column

of the table. Were the densities in the *Durchmusterung* fields exactly the same as those in the much smaller fields used for the *Mount Wilson Catalogue*, and were there no errors of observation, these deviations would be the same as those listed in Table II for $m = 16.0$, or, in the case of the Bruce telescope, equal to the mean of those for 16.0 and 18.0. But since the actual limiting magnitudes deviate from the adopted mean values by an average of 0.4 mag., there should

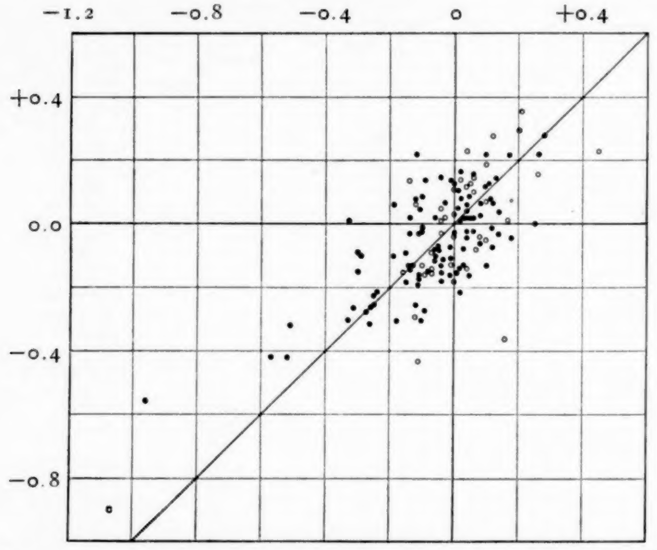

FIG. 1.—Correlation diagram. Corrections to mean distribution table (log N_m) for S.A. 1–139 as indicated by the *Mount Wilson Catalogue* (abscissae), and the *Harvard-Groningen Durchmusterung* (ordinates). Points represent observations with the Metcalf telescope; circles, with the Bruce telescope.

be a corresponding uncertainty of ± 0.12 in the interpolated log N_m, and hence in the deviations in the last column of Table III. The actual mean difference between the two series of deviations, without regard to sign, is ± 0.11. This disturbs, but does not destroy, the parallelism of the deviations, as is shown by the fact that for one hundred areas the *Durchmusterung* deviates from the mean distribution in the same direction as the *Mount Wilson Catalogue*, while for only thirty-nine areas are the departures opposite in sign. The relations are better shown by the correlation diagram, Figure 1, in which *Durchmusterung* and *Catalogue* deviations for the same

area are plotted against each other. A closer correlation would be acceptable; nevertheless, the agreement is close enough to justify the use of the *Durchmusterung* in the manner proposed.

For this purpose we assume that the mean limiting magnitude 16.86 found for the Bruce telescope from forty fields among S.A. 1–139 also applies to S.A. 140–206. The resulting deviations of these areas from the mean distribution table are in the last column of

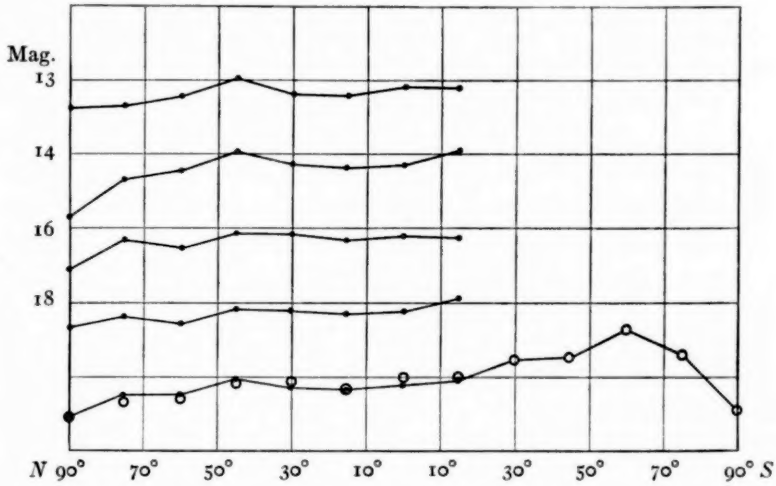

Fig. 2.—Systematic corrections to mean distribution table for zones of declination. Points are based on the *Mount Wilson Catalogue*, circles on the *Harvard-Groningen Durchmusterung*. The adopted curve at the bottom is assumed to apply to all magnitudes. Scale: Interval between axes of separate curves = 0.50 in log N_m.

Table IV. These are the only extensive data now available on the distribution of faint stars in declinations south of $-15°$.

An inspection of results for all the Selected Areas shows that the large-scale irregularities bear no simple relation to the equatorial co-ordinates of the fields. For a given declination the deviations are generally systematic; and later, in connection with the Astrographic Zones, we shall need values of these systematic departures from the mean distribution for different zones of declination. These are easily found by forming algebraic means of the deviations in Tables II and IV for all areas having the same declination. The results are given in Table VII and illustrated in Figure 2.

For the northern hemisphere these differences are systematically negative and nearly independent of magnitude. The means for the four limits, which are adopted as applying to all magnitudes in the interval $m = 13$ to 18, agree well, where comparison is possible, with the corresponding systematic differences found from the *Durchmusterung*. This gives an additional check on the reliability of the

TABLE VII

SYSTEMATIC DEVIATIONS FROM MEAN DISTRIBUTION FOR ZONES OF DECLINATION

δ	No. of Areas	m 13.0	14.0	16.0	18.0	Mean	H-G D.M.
+90°	1	−0.19	−0.43	−0.27	−0.17	−0.26	−0.27
+75	6	− .16	− .16	− .08	− .09	− .12	− .16
+60	12	− .10	− .11	− .13	− .14	− .12	− .14
+45	24	+ .01	+ .02	− .03	− .04	− .01	− .04
+30	24	− .09	− .06	− .04	− .05	− .06	− .03
+15	24	− .10	− .09	− .08	− .06	− .08	− .08
0	24	− .04	− .07	− .05	− .05	− .05	.00
−15	24	−0.05	+0.03	−0.06	+0.04	−0.01	.00
−30	24						+ .12
−45	24						+ .14
−60	12						+ .33
−75	6						+ .15
−90	1						−0.22

distribution south of −15°. The decidedly asymmetrical distribution of density in the two hemispheres shown by Figure 2 is in general agreement with the well-known richness of the southern sky.

ASTROGRAPHIC ZONES

The general features of the astrographic data have been discussed in *Contribution* No. 301, in which Table XIII gives references to all the published counts. The zones, 2° wide, are completely covered by the counts. Although rapid changes in density are much smoothed out by the large interval in right ascension used in grouping the counts, there is a compensating advantage in that the adopted densities are based on mean results for six or eight plates. This greatly lessens the influence of fluctuations in the limiting magnitude from plate to plate.

A calibration of the provisional magnitude scales, based on the

mean distribution table and on the assumption that a given scale-reading corresponds always to the same magnitude, is given in Table IV of *Contribution* No. 305. It now appears from Table VII and Figure 2 that the real average density for a zone of declination differs appreciably from the adopted mean distribution for the same zone. The magnitudes of *Contribution* No. 305 therefore require a systematic correction. This is applied directly to the deviations at a later stage in the discussion.

The observed densities have been compared with the mean distribution for three values of m as shown in Table VIII, the third, fourth, and fifth columns of which give the corresponding scale-readings or provisional magnitudes used in the published zones. The range in m for the different zones is small, and it will be sufficient to assume that the densities refer to the respective mean magnitudes for the entire series, namely, 9.24, 11.12, and 13.05 mag.

Most of the values of log N_m used in the present discussion were taken directly from the earlier reduction sheets, but a few were specially formed for new values of m in order to obtain greater uniformity in the limiting magnitudes. To avoid printing extensive tabular data, we have reluctantly decided to give only the deviations of log N_m from the mean distribution of Table XIV. The differences, observed *minus* mean, are collected in Table IX, arranged according to hours of right ascension and zones of declination. The unit is 0.01 in log N_m; the numerals I, II, and III designate the magnitude limits listed in Table VIII. These data have been condensed by combining the deviations for adjacent zones in accordance with the grouping of declinations shown by the spacing in Table IX. The means, corrected for systematic deviation depending on declination, are in Table XI.

The correction here referred to is that given in Table VII. The limiting magnitude for each Astrographic Zone is derived from the mean distribution table with the aid of the mean observed density of the zone. Since this deviates systematically from the mean distribution, the limiting magnitude, and hence the individual deviations in Table IX, will be in error, the latter systematically so.

The corrections for this zonal disturbance cannot be determined from the astrographic data at present, and the differences in Table

TABLE VIII

PROVISIONAL MAGNITUDES FOR DIFFERENT SCALE READINGS OF ASTROGRAPHIC ZONES

Zone	Observatory	Scale-reading			m		
		I	II	III	I	II	III
+62°.....	Vatican	40	20	0	8.91	11.14	12.97
+31......	Oxford	35	26	12	9.60	10.70	12.89
+30......	Oxford	45	26	12	8.66	10.86	12.92
+29......	Oxford	35	18	All	9.39	11.52	13.51
+28......	Oxford	40	18	9	8.99	11.57	13.08
+27......	Oxford	35	18	All	9.20	11.35	13.28
+26......	Oxford	35	26	All	9.32	10.34	13.48
+25......	Oxford	35	26	12	9.56	10.63	12.77
+23......	Paris	8.0	9.5	11.5	9.14	11.05	13.14
+22......	Paris	8.0	9.5	11.5	9.20	10.97	13.17
+17......	Paris	7.5	8.5	12.0	9.35	10.72	13.17
+16......	Bordeaux	7.5	9.5	12.5	8.60	11.49	12.77
+15......	Bordeaux	8.5	9.5	12.5	9.66	11.26	12.65
+14......	Bordeaux	8.5	9.5	12.5	9.96	11.52	13.00
+ 9......	Toulouse	8.5	10.5	12.3	9.06	11.05	12.55
− 1......	Algiers	7.9	8.9	11.9	9.20	10.92	12.96
− 3......	San Fernando	8.0	9.0	11.0	9.43	10.91	13.04
− 4......	San Fernando	8.0	9.0	11.5	9.14	10.76	12.96
− 5......	San Fernando	8.0	9.0	11.5	9.19	10.73	12.99
− 6......	San Fernando	8.0	9.5	All	9.30	11.34	12.90
−15......	Tacubaya	8.0	9.5	11.0	9.28	11.28	12.97
−16......	Tacubaya	8.0	9.5	11.0	9.11	11.26	13.06
−17......	Hyderabad	50	30	8	9.54	11.31	13.36
−18......	Hyderabad	50	30	12	9.60	11.43	12.79
−19	Hyderabad	50	30	12	9.32	11.36	12.90
−25......	Cordoba	41	21	11	8.68	11.41	12.89
−27......	Cordoba	41	21	1	8.61	11.44	13.04
−29......	Cordoba	31	21	1	9.27	10.76	12.98
−31......	Cordoba	31	21	1	9.14	10.65	13.20
−32......	Perth	B	E	M	9.20	10.89	13.16
−34......	Perth	B	E	M	8.92	10.89	13.22
−36......	Perth	B	D	J	9.40	10.64	13.08
−38......	Perth	40	20	10	9.33	11.34	12.89
−41......	Cape	200	110	−3	9.60	11.33	12.90
−42......	Cape	200	100	−5	9.46	11.84	13.58
−43......	Cape	200	100	−5	9.58	11.82	13.52
−65......	Melbourne	31	16	All	8.96	11.01	13.09
Means..	....	....	....	....	9.24	11.12	13.05

TABLE IX

Provisional Deviations from Astrographic Zones

(Unit = 0.01 in log N_m)

δ	$a=0^{h}.5$			$1^{h}.5$			$2^{h}.5$			$3^{h}.5$			$4^{h}.5$			$5^{h}.5$		
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
+62°......	+13	+10	+16	+29	+14	+ 8	+16	− 2	− 8	+11	0	− 8	+ 3	− 7	−15	+ 3	− 3	+ 4
+31......	−30	−37	−16	− 9	−14	− 7	−14	−20	−15	−14	−33	−45	−55	−54	−54	− 3	− 7	+ 2
+30......	−21	−35	−38	−20	−29	−16	−15	−24	−16	−49	−42	−48	−47	−63	−63	− 1	−11	−16
+29......	0	− 9	−13	+16	+13	− 3	+10	+ 3	− 8	+ 2	− 8	−14	−30	−51	−51	−16	−41	−42
+28......	− 8	−11	− 9	− 7	−16	−17	+ 5	−21	−34	+ 7	−26	−39	−33	−68	−82	−28	−43	−49
+27......	+25	+ 3	− 8	+ 8	+ 6	+ 3	+ 9	− 3	−17	−23	−17	−20	−45	−52	−61	− 7	−20	−36
+26......	−20	−18	− 7	−42	−34	−11	−13	−20	−17	0	− 8	−27	−41	−41	−61	−12	−10	−20
+25......	−14	−21	+10	− 4	−13	− 2	− 8	−12	−24	+ 5	0	−15	−30	−47	−65	− 6	+ 2	−11
+23......	+ 9	+10	+ 5	− 9	− 5	− 3	−14	−12	−13	+10	− 8	−29	−11	−37	−43	−31	−19	−18
+22......	− 3	+ 1	− 8	− 1	− 7	−27	− 8	−12	−10	0	−14	−22	−17	−28	−32	−25	−34	−27
+17......	−21	−17	−10	−26	−34	−17	−39	−36	−26	−44	−38	−28	−51	−54	−57	−39	−27	−17
+16......	−13	+ 1	0	+ 8	+ 1	− 2	− 7	−11	−18	−38	−21	−32	− 3	−34	−45	−33	− 9	−14
+15......	+26	+16	+ 7	+27	+10	+ 2	+20	+ 3	− 4	−22	−18	−21	− 6	−23	−32	− 8	−11	− 6
+14......	− 1	− 1	0	−34	−20	−26	+ 2	−15	−26	−21	−11	−10	−11	−23	−31	+11	+13	+11
+ 9......	−31	−10	− 3	−29	−24	−14	+ 5	−16	− 6	−20	−33	−11	0	−15	− 7	−10	− 8	− 3
− 1......	−35	−14	− 6	−59	−20	−15	−53	−25	−10	−19	−18	− 9	− 8	−10	−12	+10	− 4	−18
− 3......	+ 2	+ 1	+ 1	+ 8	+ 5	+ 9	+ 4	+ 3	− 4	+13	− 2	− 5	+ 8	− 3	−11	− 2	− 8	−29
− 4......	+14	0	+ 8	+ 4	− 5	+ 5	+ 4	− 3	+ 4	+ 9	− 5	− 2	+ 7	− 1	− 6	− 4	−14	−15
− 5......	+ 5	− 4	+ 2	+ 9	0	− 1	− 1	− 4	− 4	+ 9	− 1	− 3	+10	+ 2	− 9	+ 2	− 8	−15
− 6......	+ 6	+ 2	0	0	− 1	+ 3	+ 3	+ 9	+10	0	+ 7	− 7	− 1	− 4	0	0	−11	−21
−15......	+22	+23	+ 2	+15	+26	+ 8	+16	+ 2	− 9	+ 7	− 5	−15	+14	− 4	−22	+ 7	+18	− 6
−16......	+19	+22	+ 8	+26	+10	0	+17	+ 4	−12	+11	0	−22	+11	− 3	−34	+ 8	+ 2	−15
−17......	+20	+30	+15	0	+13	+ 6	− 9	+ 6	+ 3	− 9	− 9	−10	+ 3	−13	− 9	+31	+24	−12
−18......	+ 6	+14	+19	+14	+ 9	+17	+ 6	+16	+15	− 5	+11	+ 6	+14	+14	+ 7	+ 7	− 7	−10
−19......	+30	+ 8	+ 8	+22	+12	+ 7	+25	+14	+ 8	+ 7	+ 8	+ 6	+19	+13	+ 9	− 3	+ 5	+ 6
−25......	+ 3	0	+ 3	− 2	+ 9	+14	− 2	− 5	+ 4	+20	− 1	0	+ 7	− 9	− 2	+11	+ 9	+ 4
−27......	+ 1	+ 5	+15	+ 2	+ 7	+ 7	− 2	+ 2	+11	+ 9	− 2	− 8	−23	− 5	−10	+17	− 4	− 3
−29......	− 6	− 9	0	− 5	− 5	+ 2	−27	−19	− 2	− 2	+ 2	+ 1	+ 7	+ 8	+11	+ 2	+ 4	0
−31......	+ 8	+10	+ 5	+11	− 1	− 1	+12	+ 3	− 7	− 9	−13	− 8	− 1	− 5	−20	−17	−19	−28
−32......	+18	+13	+ 6	+10	+ 1	− 3	− 7	+ 1	− 2	+ 6	− 3	−15	−21	−19	−17	−20	−12	−16
−34......	−39	−20	−18	−33	−12	−13	− 2	− 7	− 8	−22	− 3	−11	+ 9	+12	+ 2	+16	+ 4	− 6
−36......	+15	+13	+10	+14	+ 5	+10	+11	0	+ 2	− 3	+ 2	− 6	−10	−10	−14	−13	−17	−19
−38......	+11	+ 9	+ 5	+19	+16	+15	+ 4	− 3	− 3	−10	− 8	−13	− 4	0	− 3	−11	− 3	−11
−41......	+ 4	+ 8	+ 7	− 7	+ 3	− 1	0	− 1	−11	+18	− 1	−19	− 4	−14	−14	+21	+ 4	+ 9
−42......	− 1	− 8	−11	+ 5	+ 2	− 5	− 5	− 3	− 5	− 8	− 3	− 9	+17	+ 3	− 4	+22	+13	+ 4
−43......	+ 6	− 4	− 2	+ 9	− 7	− 2	− 9	−14	−11	−13	− 4	− 2	− 9	−10	− 5	− 2	+ 4	+ 6
−65......	−15	− 5	−14	+19	−16	−21	− 4	−11	−23	−24	− 2	−17	+ 3	− 2	−20	+ 5	+ 6	− 6

TABLE IX—*Continued*

δ	$6^{h}.5$			$7^{h}.5$			$8^{h}.5$			$9^{h}.5$			$10^{h}.5$			$11^{h}.5$		
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
+62°	+7	+8	+12	−2	+7	+11	−19	−1	−4	−11	−3	−6	−13	+5	+4	+11	0	−4
+31	−1	−3	−14	+8	+5	+6	−25	−12	−16	+9	−5	−16	+15	+6	+8	−12	−3	−1
+30	−23	−18	−13	−41	−18	−8	−32	−18	−7	−13	0	−5	−2	0	−3	+18	−2	−1
+29	−11	−19	−10	−23	−10	0	+10	+12	+5	+5	+11	+6	+15	+18	+16	+14	+14	−4
+28	−29	−17	−6	−26	−40	−22	+3	+8	+7	−1	+17	+31	−21	+7	+12	+22	+15	+21
+27	−18	−21	−23	+3	+17	+7	+28	+5	+3	−19	−8	+1	−14	−8	+5	+7	+29	+12
+26	−14	−4	−11	−4	+3	−9	−6	−10	−13	−6	−2	−4	−5	−3	+5	−9	−11	0
+25	+2	+16	+11	+3	−1	−2	+9	+19	+8	0	+4	−8	+15	+13	+5	−4	−9	−12
+23	−6	−2	+4	−13	0	+12	−20	−8	+2	+14	+3	+1	+12	+3	−6	−2	+4	−2
+22	−7	−10	−5	−15	−6	−7	−8	+2	−3	−8	0	−5	+8	+1	−5	+7	+13	+2
+17	−27	−13	−1	+13	+12	+3	+17	+14	0	+19	+16	−4	+19	+17	−5	+16	+11	+5
+16	+11	+12	+10	+5	+7	+14	−14	+5	+3	+26	+3	−2	+43	+22	+10	+35	+15	+5
+15	+12	+14	+13	−6	+5	+18	−8	+2	+12	+12	+9	+3	+7	+8	+2	+19	+17	+7
+14	+23	+26	+33	+22	+20	+22	+15	+1	−5	+7	+2	−7	−30	−7	−5	−8	−9	−12
+9	+40	+19	+9	+22	+13	+10	−20	+1	+7	−22	−6	+4	−10	+2	−4	−14	−1	−1
−1	+3	+3	+8	+1	+21	+25	+1	+11	+17	−3	−15	−12	+16	+3	−8	+29	+14	+7
−3	−12	−7	−9	−3	+5	+9	+5	+8	+10	+6	+15	+5	+5	+6	−3	+2	+5	+4
−4	−16	−14	−9	−11	−2	+11	+3	+8	+4	−8	+9	−10	−6	+3	−3	−2	0	+3
−5	−12	−11	−9	−12	−1	+13	−3	+11	+5	−9	+3	−11	−17	−6	−15	+1	+1	−2
−6	−11	−12	−13	−16	+1	+9	+4	+3	+6	−10	−6	−12	−12	0	−6	−2	−5	−10
−15	−13	−4	+3	−14	−8	+9	+25	+14	+19	−8	−12	−14	−4	−15	−7	−6	−9	−7
−16	−13	−11	−14	−18	−13	+13	+5	0	+8	+5	−14	−7	+2	−6	−10	−3	−21	−19
−17	+12	+2	−17	−15	−8	+4	−13	−9	−6	−17	−22	−17	−26	−29	−19	−21	−24	−3
−18	−11	−16	−16	−21	−27	−12	−40	−43	−31	−37	−22	−12	−7	+7	+3	−3	+1	−8
−19	+36	+13	+4	−4	+4	+14	−36	−12	−8	−22	−11	−12	−23	−4	−13	−21	−11	−9
−25	+24	+16	+8	+7	+19	+15	+6	+16	+17	+3	+2	−3	−5	−3	−13	+11	+7	−4
−27	+24	+18	+11	+20	+17	+15	+8	+13	+8	+11	0	−10	−31	−31	−22	−10	−30	−24
−29	+14	+13	+16	+9	+3	−4	−9	−3	−8	−18	−11	−4	−14	−6	−7	+5	+2	−9
−31	+8	0	−10	+3	−11	−21	−32	−35	−2	−32	−11	−5	+7	+7	+9	−7	−5	+7
−32	−4	−7	−12	−2	−4	−11	+7	+13	+13	+11	+7	+4	+14	+4	−1	+4	+9	0
−34	+17	+8	−6	+21	+17	+7	+4	+10	+13	−9	−4	−7	−13	−2	−5	−14	−13	−10
−36	+1	−14	−24	−4	−7	−24	+5	+3	+1	+4	+1	−7	−5	−6	−6	−8	−9	−12
−38	+5	−7	−7	+18	−5	−10	+12	+2	−5	+19	+10	−2	+9	+8	+1	−4	−16	−18
−41	+5	+1	−1	+22	+14	+6	+33	+19	+4	+9	+13	+4	−8	−5	+2	+6	−11	−7
−42	+18	+4	+2	+19	+7	−4	+20	+7	−7	−13	+5	−15	−9	+11	+1	−5	−13	−12
−43	+11	+3	+1	+11	+2	+3	+24	+19	+2	−2	−1	−5	−2	−8	−8	0	−8	−4
−65	+10	+13	+5	+3	−11	−5	−6	0	−3	+24	+5	+3	+36	−1	+2	−56	−44	−15

TABLE IX—*Continued*

δ	$12^{h}.5$			$13^{h}.5$			$14^{h}.5$			$15^{h}.5$			$16^{h}.5$			$17^{h}.5$		
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
+62°	−21	− 9	− 1	−31	− 6	− 2	0	− 4	− 2	− 4	+ 1	0	+ 5	+ 1	+ 2	−12	− 5	− 8
+31	+12	+14	+13	+14	+13	+14	0	+ 6	+10	+17	+13	+ 9	+ 1	− 3	− 1	+10	+ 3	− 1
+30	+14	+ 3	+ 1	−21	− 6	+16	+ 6	+ 9	+11	+ 1	+ 6	+ 6	+11	+ 8	+15	+17	+21	+49
+29	+ 5	+ 3	+ 7	− 5	− 4	+ 5	−13	−12	+ 9	+ 2	+ 1	− 6	− 1	− 5	− 5	− 8	− 5	0
+28	+21	+23	+38	+21	+15	+25	+ 5	0	+ 3	− 5	− 4	+ 3	−14	0	+ 4	− 2	− 2	+ 3
+27	+38	+30	+16	0	− 5	− 6	+ 7	+ 1	− 1	−17	+13	−10	−15	−21	−19	−12	− 6	+ 8
+26	+30	+25	+10	+ 5	+ 4	+ 3	+20	+16	+ 6	− 8	− 7	− 5	−28	−20	0	+ 9	+11	+33
+25	+ 6	+ 9	+ 5	+43	+39	+29	+ 8	+ 6	+ 1	+ 3	− 7	−12	−38	−29	−13	+ 4	+ 6	+ 2
+23	+15	− 1	− 5	+11	+23	− 8	− 4	+ 7	− 8	+ 6	+12	−17	+ 1	+ 5	− 1	− 4	+ 2	+ 4
+22	− 1	+ 9	+15	+ 8	+13	− 3	+ 6	+12	− 3	+ 3	+14	+ 7	− 2	+ 2	−12	0	+ 3	− 2
+17	+17	+10	+17	+ 7	− 7	+ 6	+15	+11	− 3	− 3	− 8	+ 3	− 3	− 8	− 4	+28	+16	0
+16	+35	+15	+ 6	−25	−11	− 4	−25	− 4	− 4	+ 8	+ 9	+ 1	− 9	− 4	− 8	+ 9	0	+ 5
+15	− 2	− 5	− 6	−15	− 7	− 1	−14	−13	− 8	− 7	− 2	− 1	− 1	− 4	0	− 2	− 3	− 7
+14	−32	− 9	− 3	− 1	− 4	− 7	+26	+11	+ 3	− 3	− 8	− 3	+24	+13	+ 3	−18	−15	−12
+ 9	−16	−15	− 6	− 8	− 7	− 3	−11	+ 1	− 6	−29	− 5	+ 2	+17	+15	+ 7	− 3	+ 1	0
− 1	+10	+ 3	− 5	− 2	+ 3	− 3	− 6	−10	− 4	− 4	−14	−15	− 7	− 9	−16	−45	−36	−43
− 3	+ 1	− 3	+ 2	− 7	+ 5	+ 8	− 9	− 5	− 1	− 1	−13	− 9	− 3	−11	− 9	−15	−24	−23
− 4	+ 4	+ 5	+ 7	− 2	+ 6	+21	−12	− 2	− 3	−11	− 8	−16	−11	−12	−22	−15	−35	−43
− 5	+ 2	+ 8	+ 6	+11	+ 5	+ 8	0	+ 3	− 3	0	+ 3	+ 2	−18	−14	−17	−20	−41	−48
− 6	+ 6	+ 6	+ 4	+ 8	+ 1	0	+13	+ 9	+ 2	+ 7	+ 7	− 1	−25	−12	−14	−32	−44	−47
−15	+16	−14	−15	− 1	− 5	− 4	− 7	−11	0	−15	−17	− 9	−28	−23	−36	−36	−24	−35
−16	−12	− 7	− 5	− 6	− 9	−11	−15	−11	−13	−18	−21	−26	−26	−42	−33	−26	−17	−16
−17	− 6	−11	− 7	+ 9	+ 2	+ 7	+ 9	+13	+17	+ 1	− 3	+10	− 5	−18	−22	− 3	−17	−12
−18	− 3	+ 1	− 3	− 4	+ 1	−10	−18	− 3	−10	+25	+11	− 8	−25	−25	−37	−21	−24	−25
−19	−29	−18	−21	−22	−25	−16	−23	−19	−17	−18	−27	−21	−15	−45	−50	−48	−38	−35
−25	−17	− 8	0	+ 4	+ 3	− 1	−19	−16	−14	−11	−25	−22	−13	−24	−23	−24	−13	−25
−27	−12	−19	−26	−25	− 3	−19	+ 4	−11	− 1	+ 6	− 6	−17	− 2	−10	−14	−30	−10	−10
−29	−12	− 9	+ 6	−20	− 3	+11	+ 3	+ 3	+ 6	− 4	− 4	− 9	+ 4	− 2	− 8	+20	+18	+ 6
−31	+ 7	+ 9	+ 5	−13	+10	+20	−28	− 7	+15	−10	−17	−12	−10	−11	− 2	+48	+47	+ 7
−32	+ 4	0	− 6	+ 3	+ 3	0	+ 7	+ 8	+11	+ 1	− 4	− 3	+ 5	− 3	+ 2	+22	+21	+19
−34	− 5	−12	− 4	+11	+ 7	+ 6	−13	− 6	+ 6	−16	−21	−20	− 1	− 6	− 9	+37	+21	+16
−36	−33	−14	− 9	+ 7	+ 7	0	−24	− 7	− 5	−19	−13	−19	−35	−40	−35	+ 7	0	− 2
−38	−11	−17	−20	−19	− 8	− 2	−31	−12	+ 1	−26	−21	− 7	−20	−31	−37	− 4	+ 6	+10
−41	−11	− 3	+ 1	−10	+ 2	− 3	−16	−12	− 6	−18	−11	− 7	−12	−16	−24	− 8	+ 4	+12
−42	−24	−28	−26	+ 3	− 6	−10	−28	−15	−11	− 7	−13	− 4	−11	−24	−13	− 4	+ 3	+21
−43	−47	−33	−25	−17	− 8	− 3	−19	− 8	−10	− 4	− 6	−10	−29	−34	−34	− 1	0	+ 7
−65	+43	+14	+13	−16	− 8	− 5	+16	+12	−15	− 6	+14	+24	+23	+ 3	+ 5	−28	−19	− 8

TABLE IX—*Continued*

δ	$18^{\mathrm{h}}.5$			$19^{\mathrm{h}}.5$			$20^{\mathrm{h}}.5$			$21^{\mathrm{h}}.5$			$22^{\mathrm{h}}.5$			$23^{\mathrm{h}}.5$		
	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III	I	II	III
+62°	−11	+7	+2	−20	+3	+12	+23	+6	+2	+26	+7	−1	+11	−2	−12	+21	+5	0
+31	−3	−4	−16	+17	+17	+18	+13	+8	+10	−6	−3	−1	+8	0	+8	−5	−9	−10
+30	+3	−1	−3	+37	+27	+21	+35	+41	+16	+9	+1	−7	−15	−11	−16	−27	−36	−40
+29	−6	−10	+4	+37	+35	+27	−23	−7	−11	−9	−16	−12	−17	−9	−4	+15	−6	+1
+28	+35	+34	+7	−18	−7	−3	−3	−2	−9	−28	−16	−5	−21	−17	−2	−14	−24	−19
+27	−12	−5	+18	+8	+4	+6	+8	+8	+7	−19	−5	+3	−17	−19	+5	+1	−13	−11
+26	−3	+2	+7	+16	+11	−4	+24	+18	+17	+5	+5	+9	−13	−16	−6	+2	−5	−14
+25	−11	−14	+4	−10	−14	−10	−18	−13	+2	−10	−7	+5	−7	−10	+25	−24	−20	−10
+23	−6	−12	−6	−5	−11	−13	−8	−10	+1	−10	−12	−6	−1	+11	+9	+13	+3	+5
+22	−10	−11	−2	−13	−5	−14	+4	+5	+24	0	+2	+9	−10	+2	+9	+3	−2	−13
+17	−5	−7	−9	−10	−12	−15	−8	−5	+2	−39	−24	0	−36	−34	−1	−30	−35	−4
+16	+12	−1	−2	−26	−16	−5	−13	−4	+7	−32	−6	+3	−19	+7	+11	+10	−1	0
+15	−10	−15	−15	−18	−17	−19	+10	+6	+11	+12	+12	+7	+10	+12	+9	+7	+6	+2
+14	−10	−12	−13	−16	−7	0	−8	−5	+8	−1	+4	−4	+3	+8	+2	−11	−7	−7
+9	+21	+23	+12	+5	−1	−2	+45	+33	+20	+3	+1	+1	+29	+19	+13	+13	+5	−1
−1	−39	−44	−69	−10	−22	−19	−2	+8	+13	+15	+11	+6	+9	−9	−13	−3	−10	−15
−3	−17	−35	−51	−16	−13	−8	−7	−2	+23	−4	−2	+5	−3	+1	+13	−7	+1	+11
−4	−26	−34	−57	−14	−19	−29	−4	−1	+15	+2	−2	+	+1	+3	0	0	−4	+10
−5	−21	−32	−30	−11	−14	−14	+2	−4	+17	+7	+1	+12	−5	+4	+5	−1	+2	+6
−6	−25	−35	−23	−15	−19	−15	+4	−15	+8	−2	−3	+8	+4	−1	+2	+2	−1	+9
−15	−18	−3	+4	−2	+4	+10	+9	+10	+13	+27	+11	+10	+23	+11	+4	+22	+11	+25
−16	−9	−5	+2	−18	+4	+9	+8	+13	+18	−5	+5	+9	+9	+7	−1	+23	+12	+1
−17	−29	−31	−16	−21	−39	−1	−6	+2	+7	+12	+22	+3	+10	+9	+7	+1	+17	+8
−18	+13	−4	−1	−28	−33	−19	+9	−5	−2	+24	+12	+13	+6	+10	+13	+3	+5	+6
−19	−14	−13	−11	+3	−1	+1	−15	+8	+10	−1	+17	+11	+2	+3	+7	+20	+22	+13
−25	−3	−1	−2	−1	+4	+11	0	−18	+13	+10	+2	+3	−8	−12	−7	+5	−3	+4
−27	+3	+8	+11	+2	+5	+6	−13	−10	+4	+7	+1	−4	−9	−8	−1	−22	−20	−3
−29	+24	+23	+18	+17	+16	+4	+20	+3	−4	+13	+13	0	−15	−9	−7	+20	+8	−1
−31	+34	+40	+29	−26	−6	+23	−24	−10	−1	0	−7	−6	−10	−1	0	−1	+2	+9
−32	+22	+14	+23	+12	+4	+5	−9	−4	+6	−16	−19	−13	−25	−13	−14	+14	+27	+12
−34	+32	+19	+21	+5	+6	+4	+2	+4	+8	−8	−4	+3	+4	+1	+3	−10	−2	+6
−36	+18	+18	+20	−15	−14	−1	+2	−1	+8	+4	+12	+20	0	+2	+8	0	+10	+11
−38	−5	−5	+11	−12	−11	0	−7	+7	+11	+10	+1	−4	−9	+1	+6	+9	+15	+8
−41	−16	+2	+23	−17	−7	+11	−15	−7	+6	−19	−18	−16	+16	+2	+1	+27	+14	+2
−42	+21	+19	+48	+5	+7	+19	+3	+1	+6	−11	−2	+5	−4	−4	−8	−2	−10	−8
−43	+15	+21	+24	0	+7	+13	+12	+5	+7	0	−1	−2	−1	−10	−17	+8	−5	−3
−65	−1	+13	+25	+9	−25	−24	−12	−13	−3	−4	−5	+4	−7	−16	−13	−16	−27	−32

IX must be corrected with the aid of Table VII, based on the Selected Areas. This table is directly applicable to the fainter astrographic stars, which are of the thirteenth or fourteenth magnitude; but it is doubtful if it is equally applicable to the stars of the ninth and eleventh magnitudes. The small dependence on magnitude shown by Table VII and Figure 2 indicates, however, that it is safer to use the correction found from the faint stars than to omit it altogether, and this procedure has been followed; but the results, for the ninth magnitude especially, are much less reliable.

The corrections were applied to the means of the deviations in Table IX, the values used being those in Table X, which correspond

TABLE X

SYSTEMATIC CORRECTIONS USED FOR ASTROGRAPHIC ZONES

δ	Corrn.	No. Zones	δ	Corrn.	No. Zones
$+62°$	-0.12	3	$-17°$	$+0.01$	5
$+29$	$-\ .06$	5	-28	$+\ .10$	4
$+24$	$-\ .07$	4	-35	$+\ .13$	4
$+14$	$-\ .08$	5	-42	$+\ .14$	3
$-\ 4$	-0.04	5	-65	$+0.27$	1

to the mean declinations in question. Thus from Table IX, zones $+31°$ to $27°$, $\alpha = 0^{h}.5$, and limit I, the mean difference is -0.07. The correction for mean declination $+29°$, from Table X, is -0.06. The corrected mean difference -0.13 appears in the second line of the second column of Table XI.

In estimating the reliability of the results in Table XI, it should be noted that the published counts for $+62°$ are really means for the three zones $+64°$, $+62°$, and $+60°$.[1] The deviations for $-65°$, based on the single Melbourne zone, are of relatively low weight. Those for $+24°$, $+14°$, $-4°$, and $-17°$ have the highest weight, since they include four or five zones, distributed among two or more observatories.

SUPPLEMENTARY ASTROGRAPHIC ZONES

Table XI gives the results for the thirty-nine Astrographic Zones for which complete counts have been published. Catalogues for

[1] *Monthly Notices, R.A.S.*, **75**, 601, 1915.

TABLE XI

ADOPTED MEAN DEVIATIONS FOR ASTROGRAPHIC ZONES
(Unit = 0.01 in log N_m)

δ	$\alpha=0^h.5$			$\alpha=1^h.5$			$\alpha=2^h.5$		
	I	II	III	I	II	III	I	II	III
+62°	−5	−8	−2	+11	−4	−10	−2	−20	−26
+29	−13	−24	−23	−8	−14	−14	−7	−19	−24
+24	−14	−14	−7	−21	−22	−18	−18	−21	−23
+14	−16	−10	−9	−19	−21	−19	−12	−23	−24
−4	−6	−7	−3	−12	−8	−4	−13	−8	−5
−17	+20	+20	+11	+16	+15	+9	+12	+9	+2
−28	+12	+12	+16	+12	+12	+16	+5	+5	+12
−35	+14	+17	+14	+15	+15	+15	+15	+11	+10
−42	+17	+13	+12	+16	+13	+11	+9	+8	+5
−65	+12	+22	+13	+46	+11	+6	+23	+16	+4
	$\alpha=3^h.5$			$\alpha=4^h.5$			$\alpha=5^h.5$		
+62	−7	−18	−26	−15	−25	−33	−15	−21	−14
+29	−21	−31	−39	−48	−64	−68	−17	−30	−34
+24	−3	−15	−30	−32	−45	−57	−25	−22	−26
+14	−37	−32	−28	−22	−38	−42	−24	−16	−14
−4	−2	−8	−9	−1	−7	−12	−3	−13	−24
−17	+3	+2	−6	+13	+2	−9	+11	+9	−6
−28	+14	+6	+6	+8	+7	+5	+13	+8	+3
−35	+6	+10	+2	+7	+9	+5	+6	+6	0
−42	+13	+11	+4	+15	+7	+6	+28	+21	+20
−65	+3	+25	+10	+30	+25	+7	+32	+33	+21
	$\alpha=6^h.5$			$\alpha=7^h.5$			$\alpha=8^h.5$		
+62	−11	−10	−6	−20	−11	−7	−37	−19	−22
+29	−22	−22	−19	−22	−15	−9	−9	−7	−8
+24	−13	−7	−7	−14	−8	−9	−13	−6	−9
+14	+4	+4	+5	+3	+3	+5	−10	−3	−5
−4	−14	−12	−10	−12	+1	+9	−2	+4	+4
−17	+3	−2	−7	−13	−9	+7	−11	−9	−3
−28	+28	+22	+16	+20	+17	+11	+3	+8	+14
−35	+18	+8	+1	+21	+13	+3	+20	+20	+18
−42	+25	+17	+15	+31	+22	+16	+40	+29	+14
−65	+37	+40	+32	+30	+16	+22	+21	+27	+24
	$\alpha=9^h.5$			$\alpha=10^h.5$			$\alpha=11^h.5$		
+62	−29	−21	−24	−31	−13	−14	−7	−18	−22
+29	−10	−3	−3	−7	−1	+2	+4	+5	−1
+24	−7	−6	−11	+1	−3	−7	−9	−8	−10
+14	0	−3	−9	−2	0	−8	+2	−1	−7
−4	−9	−3	−12	−7	−3	−11	+2	−1	−4
−17	−15	−15	−11	−11	−8	−8	−10	−12	−8
−28	+1	+5	+4	−1	+2	+2	+10	+4	+2
−35	+19	+17	+10	+14	+14	+10	+7	+6	+3
−42	+12	+20	+9	+8	+13	+12	+14	+3	+6
−65	+51	+32	+30	+63	+26	+29	−29	−17	+12

TABLE XI—*Continued*

δ	$\alpha=12^{\mathrm{h}}5$			$\alpha=13^{\mathrm{h}}5$			$\alpha=14^{\mathrm{h}}5$		
	I	II	III	I	II	III	I	II	III
+62°	−39	−27	−19	−49	−24	−20	−18	−22	−20
+29	+12	+ 9	+ 9	− 4	− 3	+ 5	− 5	− 5	0
+24	+ 5	+ 3	− 1	+10	+13	− 2	+ 1	+ 3	− 8
+14	− 8	− 9	− 6	−16	−15	−10	−10	− 7	−12
− 4	+ 1	0	− 1	− 2	0	+ 3	− 7	− 5	− 6
−17	− 6	− 9	− 9	− 4	− 6	− 6	−10	− 5	− 4
−28	+ 2	+ 3	+ 6	− 4	+12	+13	0	+ 2	+12
−35	+ 2	+ 2	+ 3	+13	+15	+14	− 2	+ 9	+17
−42	−13	−17	− 3	+ 6	+10	+ 9	− 7	+ 2	+ 5
−65	+70	+41	+40	+11	+19	+22	+43	+39	+12
	$\alpha=15^{\mathrm{h}}5$			$\alpha=16^{\mathrm{h}}5$			$\alpha=17^{\mathrm{h}}5$		
+62	−22	−17	−18	−13	−17	−16	−30	−23	−26
+29	− 6	0	− 6	−10	−10	− 7	− 5	− 4	+ 6
+24	− 6	− 4	−14	−24	−17	−13	− 5	− 1	+ 2
+14	−15	−11	− 8	− 2	− 6	− 8	− 5	− 8	−11
− 4	− 6	− 9	−12	−17	−16	−20	−29	−40	−45
−17	− 4	−10	−10	−19	−30	−35	−26	−23	−24
−28	+ 5	− 3	− 5	+ 5	− 2	− 2	+14	+20	+ 4
−35	− 2	− 2	+ 1	0	− 7	− 7	+29	+25	+24
−42	+ 4	+ 4	+ 7	− 3	−11	−10	+10	+16	+27
−65	+21	+41	+51	+50	+30	+32	− 1	+ 8	+19
	$\alpha=18^{\mathrm{h}}5$			$\alpha=19^{\mathrm{h}}5$			$\alpha=20^{\mathrm{h}}5$		
+62	−29	−11	−16	−38	−15	− 6	+ 5	−12	−16
+29	− 3	− 3	− 4	+10	+ 9	+ 8	0	+ 4	− 3
+24	−15	−16	− 6	−10	−12	−17	− 7	− 7	+ 4
+14	− 6	−10	−13	−21	−19	−16	− 3	− 3	+ 2
− 4	−30	−40	−50	−17	−21	−21	− 5	− 7	+11
−17	−11	−10	− 3	−12	−12	+ 1	+ 2	+ 7	+10
−28	+24	+28	+24	+ 8	+15	+21	+ 6	+ 1	+13
−35	+30	+25	+32	+11	+ 9	+15	+10	+15	+21
−42	+21	+28	+46	+10	+16	+28	+14	+14	+20
−65	+26	+40	+52	+36	+ 2	+ 3	+15	+14	+24
	$\alpha=21^{\mathrm{h}}5$			$\alpha=22^{\mathrm{h}}5$			$\alpha=23^{\mathrm{h}}5$		
+62	+ 8	−11	−19	− 7	−20	−30	+ 3	−13	−18
+29	−17	−14	−10	−18	−17	− 8	−12	−24	−22
+24	−11	−10	− 3	−15	−11	+ 2	− 9	−13	−15
+14	−19	−11	− 7	−11	− 6	− 1	−10	−14	−10
− 4	0	− 3	+ 3	− 3	− 4	− 3	− 6	− 6	0
−17	+12	+14	+10	+11	+ 9	+ 7	+15	+14	+12
−28	+18	+12	+ 8	0	+ 2	+ 6	+10	+ 7	+12
−35	+11	+11	+15	+ 5	+11	+14	+16	+25	+22
−42	+ 4	+ 7	+10	+18	+10	+ 6	+25	+14	+11
−65	+23	+22	+31	+20	+11	+14	+11	0	− 5

other zones, however, are already in print; and although detailed counts have not been made, we have used the total number of stars observed in each hour of right ascension to the limiting magnitude of the catalogue. In several instances these supplementary data fill important gaps in declination, as, for example, the Greenwich zones, extending from $+64^\circ$ to $+90^\circ$.

The mean limiting magnitudes of the zones have been determined as usual by comparison with the mean distribution in Table XIV. Owing to the high declination, which necessitates some care in determining the areas covered by the counts, the values of $\log N_m$ for the Greenwich zones have been derived from the formula

$$\log N_m = \log N - \log \left(\frac{pA}{P}\right) ,$$

where N is the number of stars and p the number of plates in the right-ascension interval (usually one hour); and where A is the number of square degrees and P the total number of plates in the entire zone. A small region near the pole (86°–90°) has not been used. Otherwise, the treatment is the same for the zones already discussed.

The results for forty-six zones are collected in condensed form in Tables XII*a*–*e*.[1] The mean limiting magnitudes for the combined zones are at the bottom of the tables, opposite the designation m. The tabulated values of O – C have already been corrected for the systematic zone deviation of Table VII. With an exception presently to be mentioned, the correction used is given at the foot of each column. The corresponding change in magnitude, applied to m, gives the corrected limiting magnitude m'.

Were the observational data strictly homogeneous, the application of the proper systematic corrections would make all the values of m' for any given observatory sensibly equal to one another. For the Greenwich zones from 86° to 76° this is approximately the case, but it is by no means true for zones $+76^\circ$ to $+64^\circ$. In consequence,

[1] The counts for the Vatican zones discussed in the preceding section are means for the three zones $+64^\circ$, $+62^\circ$, and $+60^\circ$. Totals for the separate zones were formed from the catalogues, along with those for the other Vatican zones now available, and are here included with the supplementary data in Table XII*c*. To equalize the weight for the Vatican groups, overlapping means have been formed.

we must assume either that the limiting magnitude of the photographs steadily falls off in this region to the extent of nearly a magni-

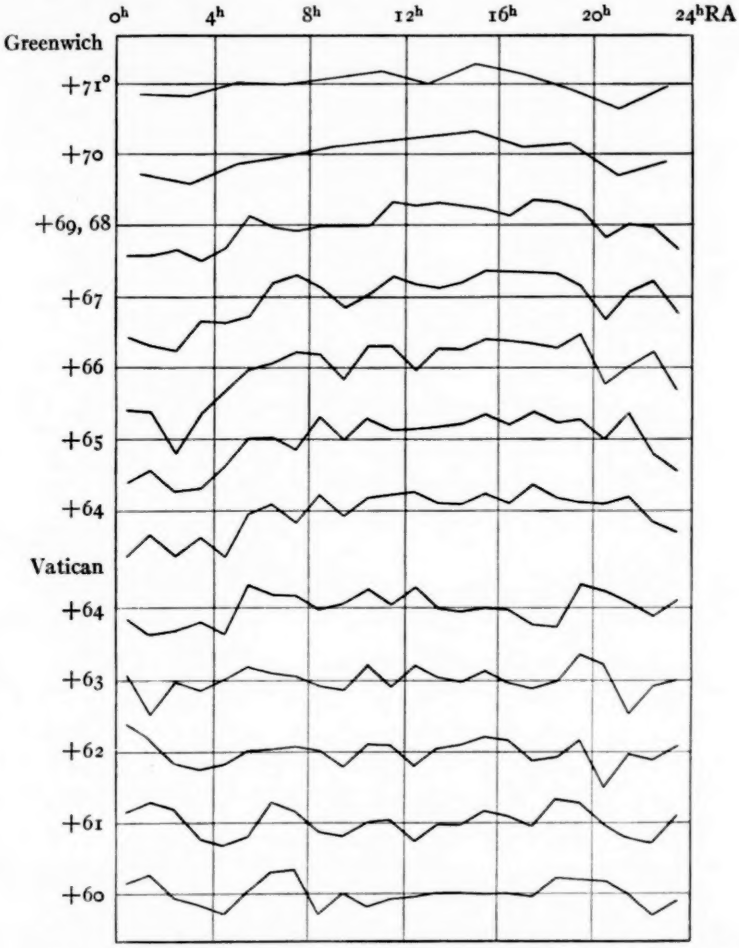

FIG. 3.—Deviations from mean distribution indicated by Greenwich and Vatican Astrographic Zones, $\delta=+60°$ to $+71°$. Scale: Interval between axes=0.60 in log N_m. The minimum centered at $\alpha=2^h$, $\delta=+66°$, which arises from an obscured region north of γ Cassiopeiae, leads to a declination-zone correction considerably larger than would be inferred from the data of Table VII.

tude or that the systematic zone deviations differ appreciably from those interpolated from Table VII. Since the Greenwich observations, which began with the 64° zone and progressed toward the pole,

extended over a period of several years, the limiting magnitude may have changed, although a change of a magnitude is unlikely. On the other hand, the corrections in Table VII are determined only at 15° intervals in declination; the possibility of rapid changes in the correction with declination is not excluded, and interpolated values for intermediate declinations may be seriously in error. The adjoining Vatican zones also show a relatively low limiting magnitude at +64°, which suggests that the interpolated corrections for this region are really too small. A plot of the original deviations for a number of

TABLE XIIa

DEVIATIONS FOR SUPPLEMENTARY ZONES

α	Greenwich		
	86°, 85°	84°, 83°, 82°	81°, 80°, 79°
$1^h.5$	−34	−23	−13
4.5	−20	−18	−29
7.5	−13	−23	−16
10.5	−24	−27	−15
13.5	−30	−23	−16
16.5	−28	−19	− 8
19.5	− 6	− 7	−15
22.5	−24	−12	−24
Corr.	−22	−19	−17
m	13.36	13.51	13.60
m'	13.96	14.03	14.06

these zones, uncorrected for systematic departure depending on declination, is shown in Figure 3. The curves are much influenced by accidental errors, but show a general similarity in the distribution for neighboring zones. The deep minimum in the first quadrant of right ascension which is so conspicuous in the Greenwich zones from 64° to 70° appears in the Vatican zone at +64°, and is probably responsible for most of the apparent variation in the limiting magnitude. The region is in the Milky Way, 6° or 7° north of γ Cassiopeiae, and appears on small-scale photographs as an area of low density.[1] There can be little doubt, therefore, that the correction depending on declination is considerably larger in this region than would be inferred from Table VII.

[1] See, for example, *Harvard Annals*, **80**, No. 4, Plate 5.

Revised corrections were computed on the assumption of constant limiting magnitudes: 13.80 for Greenwich and 13.46 for the Vatican. These values agree reasonably well with the limits from the other zones, and satisfy the conditions that the corrections applied to the Greenwich and Vatican observations of the 64° zone

TABLE XII*b*

DEVIATIONS FOR SUPPLEMENTARY ZONES

α	Greenwich			
	78°, 77°, 76°	75°, 74°	73°, 72°	71°, 70°
1^h0	−26	−28	−26	−26
3.0	−16	−24	−25	−32
5.0	−18	−13	−16	−18
7.0	−14	− 1	− 9	−16
9.0	− 9	− 8	0	− 8
11.0	−15	−21	− 7	− 4
13.0	−13	− 5	0	− 7
15.0	−11	− 2	0	+ 4
17.0	− 2	− 2	− 1	− 6
19.0	− 2	+ 5	− 8	−12
21.0	−32	−35	−44	−34
23.0	−15	−19	−26	−18
Corr.	−14	−12	−12	−12
m	13.66	13.51	13.44	13.42
m'	14.04	13.83	13.76	13.74

must be the same. The zones affected and the revised zone corrections for log N_m are:

δ	+73°, 72°	71°, 70°	69°, 68°	67°, 66°	65°, 64°	64°, 63°, 62°	62°, 61°, 60°
Rev. corr.	−13	−14	−21	−30	−36	−21	−16

These corrections, and not the values in the corresponding columns of Tables XII*b* and *c*, have been used in forming the tabulated deviations.

The limiting magnitudes in Tables XII are approximately the same as the lowest limit for the thirty-nine zones listed in Table VIII. The only exception is for the Uccle zones,[1] Table XII*d*, for which the mean limit is 15.22, or, with systematic correction for

[1] *Annales de l'Observatoire Royal de Belgique* (3d ser.), **2**, 1924. These observations are not a part of the adopted program for the *Astrographic Catalogue*, but were made by similar methods and are conveniently discussed with the Astrographic Zones. The Potsdam observations for zones +39° to +32°, *Photographische Himmelskarte*, **1–7**, 1899–1915, unfortunately were not available.

TABLE XIIc

DEVIATIONS FOR SUPPLEMENTARY ZONES

α	Greenwich			Vatican		
	69°, 68°	67°, 66°	65°, 64°	64°, 63°, 62°	62°, 61°, 60°	60°, 59°, 58°
$0^h.5$	−47	−64	−71	−14	− 2	+ 8
1.5	−47	−70	−59	−35	− 2	+12
2.5	−42	−88	−76	−31	−17	− 6
3.5	−51	−58	−67	−32	−29	−19
4.5	−40	−52	−67	−32	−32	−20
5.5	−13	−40	−38	−10	−19	−12
6.5	−24	−22	−31	−14	− 4	0
7.5	−26	−15	−44	−15	− 5	+ 3
8.5	−22	−20	−20	−23	−24	−16
9.5	−22	−39	−39	−27	−24	− 5
10.5	−22	−20	−21	− 9	−17	− 8
11.5	− 2	−12	−24	−20	−15	− 7
12.5	− 5	−27	−24	−16	−27	− 8
13.5	− 3	−18	−27	−19	−16	− 8
14.5	− 6	−16	−26	−21	−15	−11
15.5	− 8	− 6	−18	−14	− 9	−17
16.5	−13	− 8	−26	−20	−11	−14
17.5	0	−10	−12	−30	−21	−16
18.5	− 2	−12	−24	−28	− 7	− 1
19.5	− 9	−12	−23	− 4	− 3	−10
20.5	−32	−46	−32	−22	−23	−32
21.5	−20	−27	−17	−29	−22	−37
22.5	−24	−16	−46	−27	−30	−25
23.5	−42	−48	−58	−17	−14	−21
Corr.	−12	−12	−12	−12	−12	−11
m	13.23	12.98	12.84	12.90	13.03	13.16
m'	13.55	13.30	13.16	13.22	13.35	13.46

TABLE XIId

DEVIATIONS FOR SUPPLEMENTARY ZONES

Uccle +39°, 37°, 35°, 33°

α	O−C	α	O−C	α	O−C
$0^h.7$	− 8	$10^h.3$	−17	$17^h.5$	+ 1
2.6	− 7	11.7	− 8	18.7	+15
3.9	−29	12.5	−14	19.5	+10
5.6	−16	13.6	− 7	20.8	+ 6
6.6	+ 2	14.5	− 7	21.6	+ 6
7.7	− 8	15.2	− 2	22.6	− 2
9.5	+ 5	16.4	+ 2	23.4	0
Corr.					− 4
m					15.22
m'					15.33

declination, 15.33. With this exception the supplementary data may therefore be combined with those for limit III of the other zones.

TABLE XII*e*

DEVIATIONS FOR SUPPLEMENTARY ZONES

α	Toulouse +7°	Algiers 0°, −2°	Hyderabad −20°, 21°	Cape −44°, 45°, 46°, 47°	Cape, Sydney −48°, 50°, 52°
$0^h.5$	−20	+ 3	+ 9	+14	+24
1.5	−11	− 5	+ 8	+11	+19
2.5	−18	− 3	+ 2	0	+ 9
3.5	−29	− 9	− 2	− 1	+ 4
4.5	−16	−15	0	+16	+20
5.5	−20	− 7	0	+12	+13
6.5	+ 8	+17	+ 8	+23	+17
7.5	+15	+29	+21	+23	+22
8.5	+13	− 3	+ 8	+19	+ 9
9.5	−15	−11	+12	+ 3	+19
10.5	− 1	+ 1	+16	+15	+28
11.5	− 5	− 2	+ 8	+ 5	+18
12.5	− 5	− 2	+ 1	− 3	+16
13.5	− 9	+ 2	− 4	+11	+10
14.5	− 2	− 2	− 5	+13	+ 7
15.5	− 2	−19	−21	+11	+24
16.5	−11	−16	−44	+23	+19
17.5	+ 5	−44	−30	+34	+20
18.5	−22	−47	+ 3	+45	+44
19.5	+ 3	−12	+29	+24	+33
20.5	+10	+ 3	+22	+22	+26
21.5	+11	+ 9	+20	+19	+33
22.5	− 2	+ 3	+10	+12	+18
23.5	−20	+ 3	+ 9	+12	+26
Corr.	− 6	− 5	+ 4	+15	+20
m	12.77	12.78	13.52	13.56	13.47
m'	12.94	12.92	13.41	13.15	12.93

DISCUSSION

Table XIII indicates the material available for further discussion and the numbers of the tables in which the deviations from the mean distribution are collected. Those derived from the *Harvard-Groningen Durchmusterung* for S.A. 1–139, which are given in Table III, are not listed, because they provide only an intermediate step in the utilization of the *Durchmusterung* counts for the remaining Selected Areas and do not appear elsewhere in the discussion.

The data for different values of m must now be combined into tables of smoothed deviations from the mean distribution. Further, as a matter of convenience and because the systematic irregulari-

ties may be expected to bear some relation to the Milky Way, the adopted deviations must be referred to galactic co-ordinates.

The tabular deviations for each m were plotted in equatorial co-ordinates on large-scale Aitoff projection charts. Equal-density curves were drawn on superimposed tracing sheets for intervals of 0.10 in log N_m. The tracing sheets were then placed over a galactic co-ordinate net, from which the adopted deviations were read for equidistant intervals of galactic latitude and longitude. Because of the small fields used for the Selected Areas, the resulting deviations for $m=13$ and 14 are of low weight. The means for these limits as given in Table II have accordingly been combined with the deviations for the faintest limit of the Astrographic Zones. The equal-

TABLE XIII

SUMMARY FOR ADOPTED DEVIATIONS

Source	Regions	Limiting m	No. Dev. for Each m	Collected Deviations
M.W. Catalogue.........	S.A. 1–139	13, 14, 16, 18	139	Table II
Harv.-Gr. D.M...........	S.A. 140–206	16.86	67	Table IV
39 Astrogr. Zones........	10 mean δ's	9.2, 11.1, 13.0	240	Table XI
46 Suppl. Astrogr. Zones..	19 mean δ's	13–14	357	Tables XII*a–e*

density curves thus found are assumed to refer to the magnitude limit 13.5. Further, since the only data for faint stars south of declination $-15°$ are from the *Harvard-Groningen Durchmusterung* for the limit $m=16.86$, it has been necessary to use the corresponding deviations with those from the *Mount Wilson Catalogue* ($\delta=+90°$ to $-15°$) for both $m=16.0$ and 18.0.

The curves of equal density, referred to equatorial co-ordinates, are reproduced in Figures 4–8. The adopted deviations for 10° intervals of galactic co-ordinates are collected in Tables XV*a–e*, and illustrated in Figures 9–13. The general similarity of the diagrams indicates that the characteristic features of the distribution change slowly with magnitude, whence it may be assumed that the results refer to the rounded limits 9.0, 11.0, 13.5, 16.0, and 18.0. It should be remembered, however, that the deviations in the interval $\delta=-30°$ to $-90°$ for $m=16.0$ and 18.0 are the same and really refer to the limit 16.86. The corresponding portions of the density curves are therefore essentially the same.

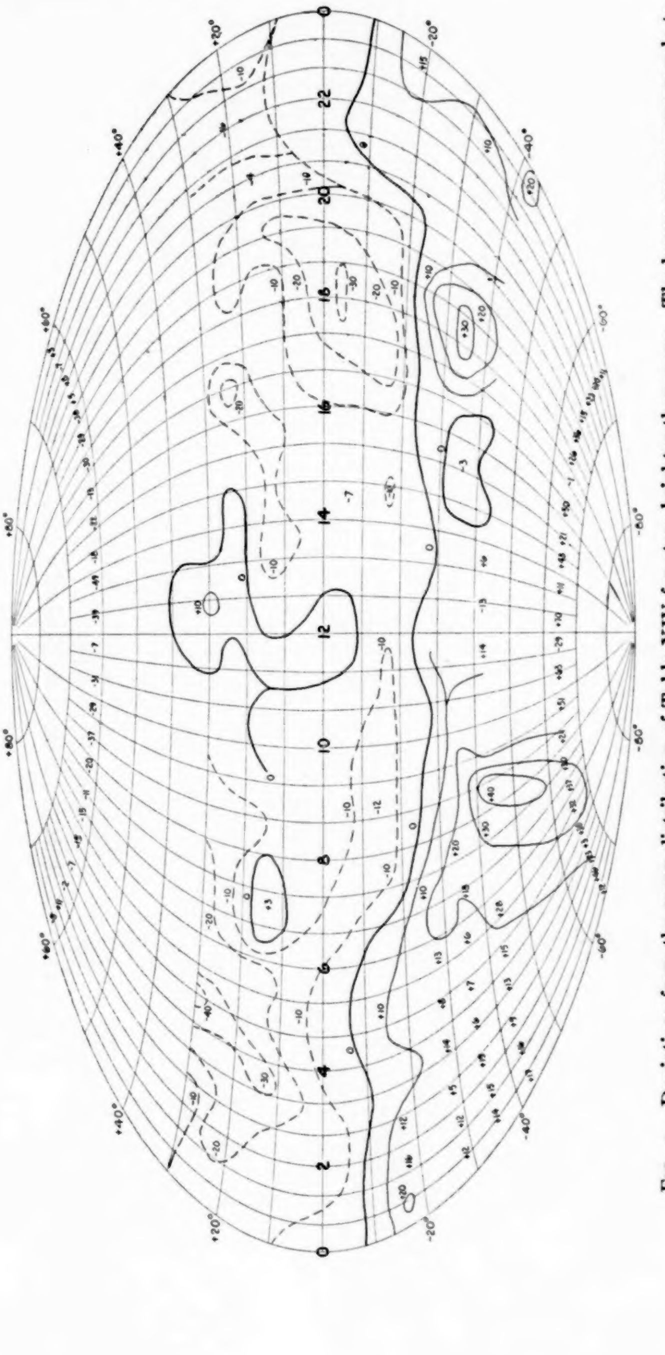

FIG. 4.—Deviations from the mean distribution of Table XIV for stars brighter than $m=9$. The heavy curve corresponds to zero deviation. Full-line curves represent positive values of $\Delta \log N_m$; dotted curves, negative values. The interval between curves is 0.10 in $\log N_m$, and the unit for the figures in the diagram is 0.01. The co-ordinates are right ascension (horizontal axis) and declination. The contours for this limiting magnitude are fragmentary and of low weight.

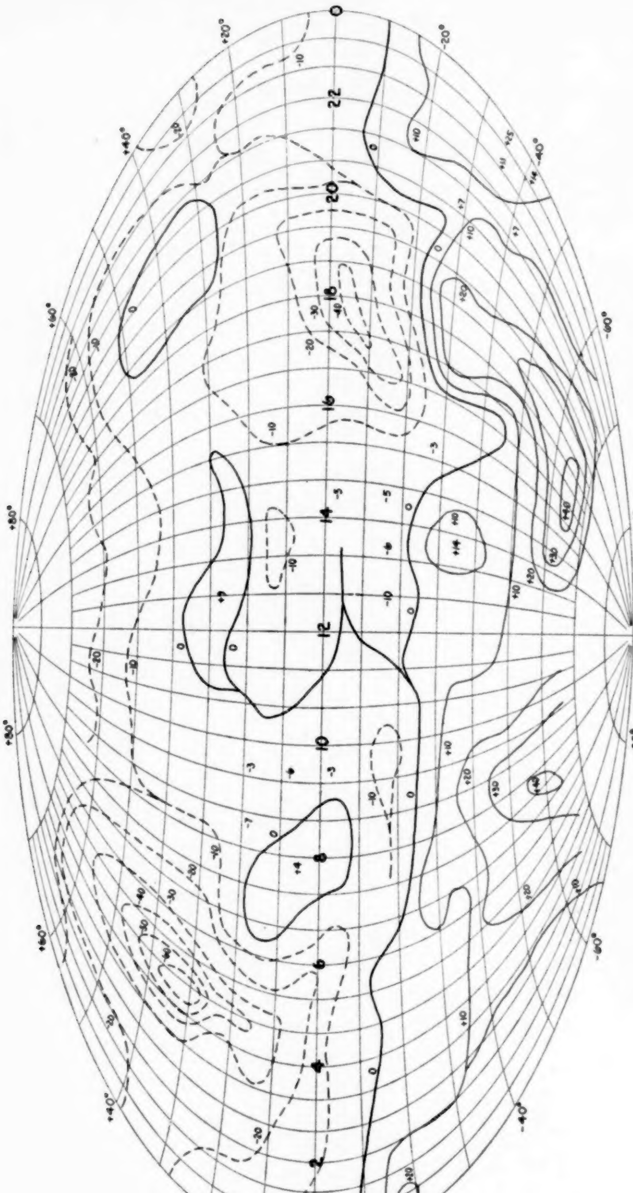

FIG. 5.—Deviations from the mean distribution for stars brighter than $m=11$

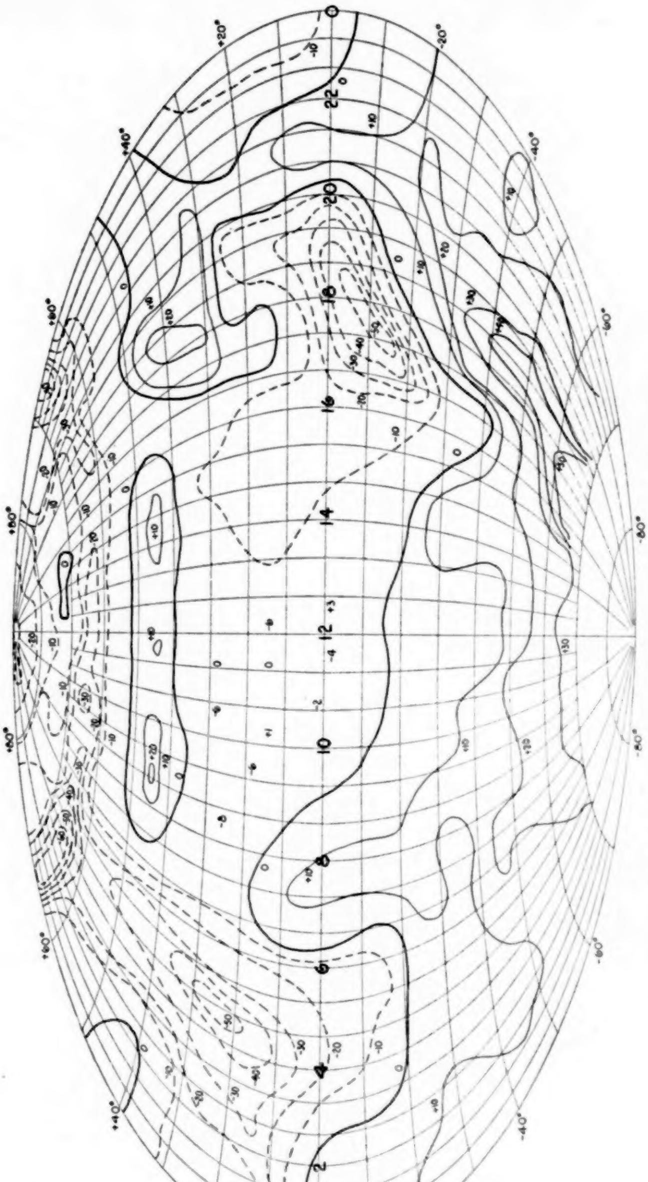

FIG. 6.—Deviations from the mean distribution for stars brighter than $m = 13{\cdot}5$

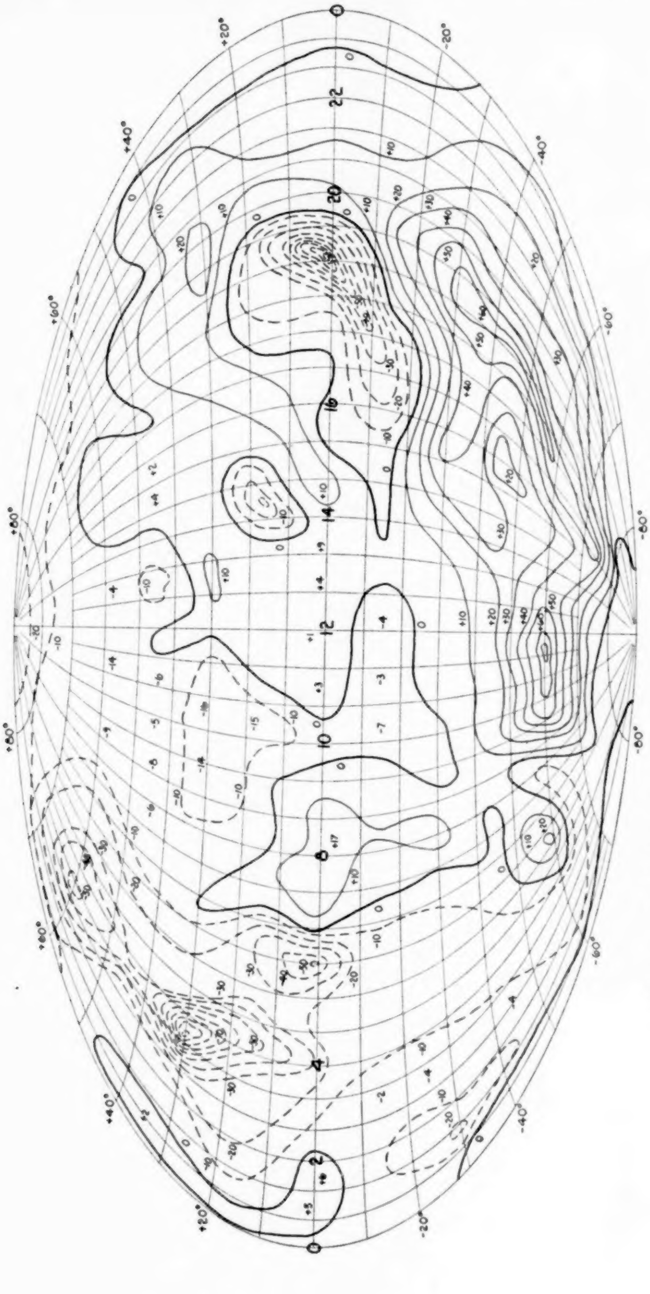

FIG. 7.—Deviations from the mean distribution for stars brighter than $m=16$

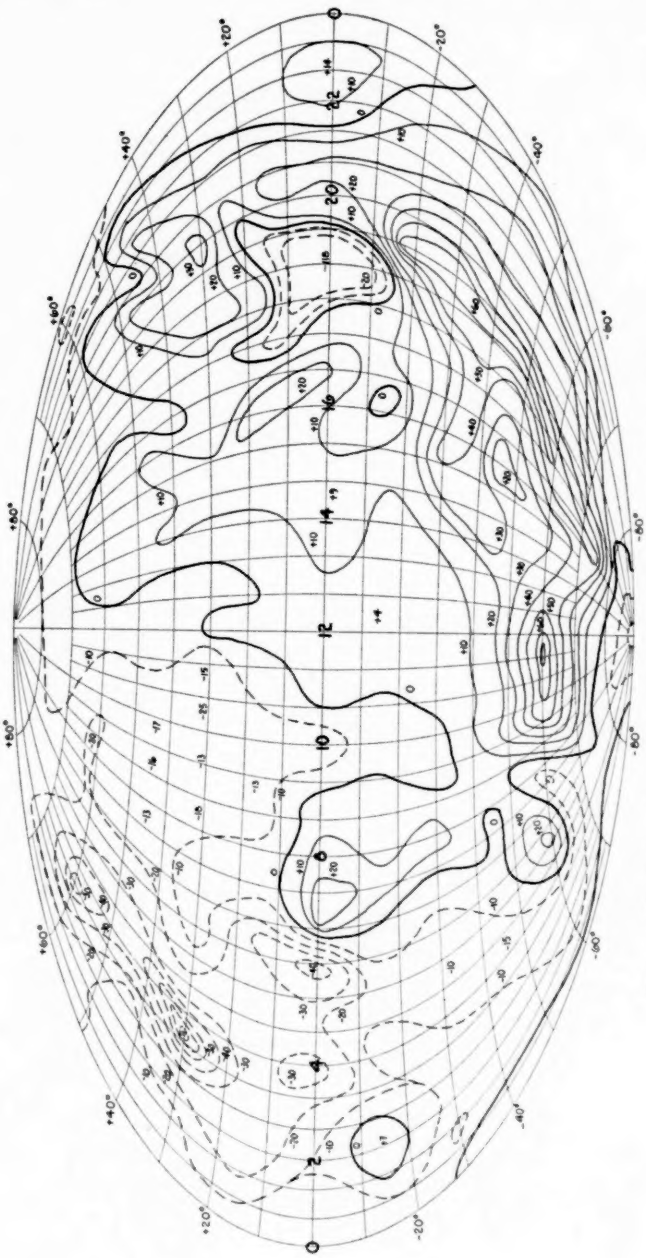

FIG. 8.—Deviations from the mean distribution for stars brighter than $m = 18$

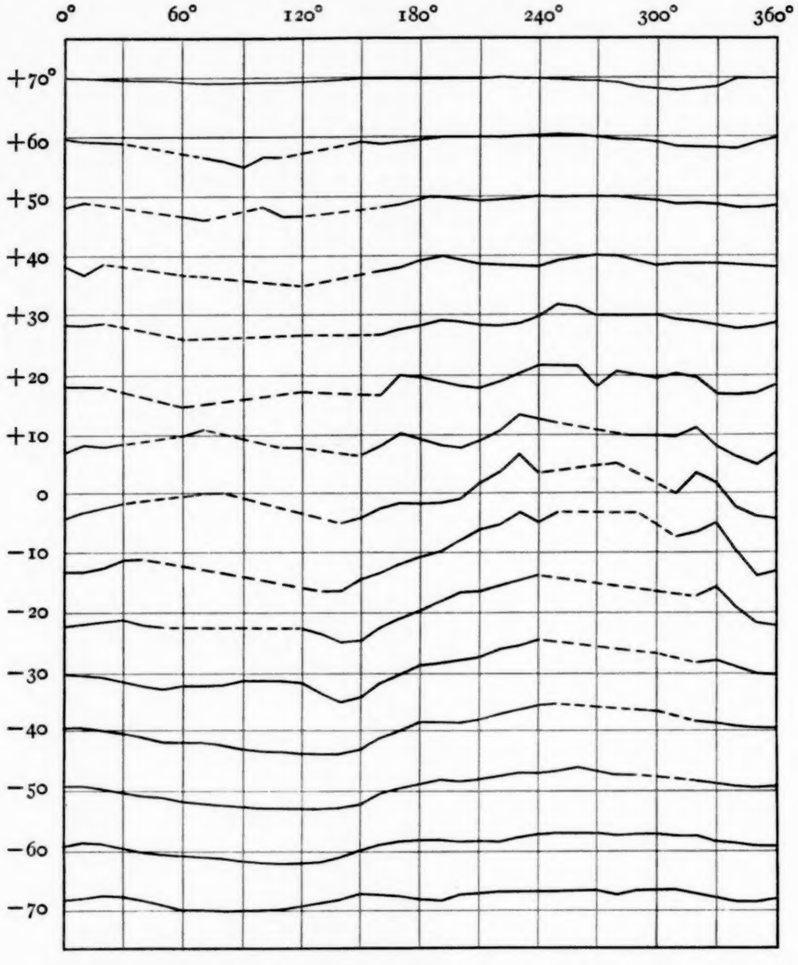

FIG. 9.—Adopted corrections (Table XV) to mean distribution (log N_m, Table XIV) for $m=9$. The abscissae are longitudes; the curves refer to the latitudes shown at the left. Scale: Interval between axes = 0.60 in log N_m.

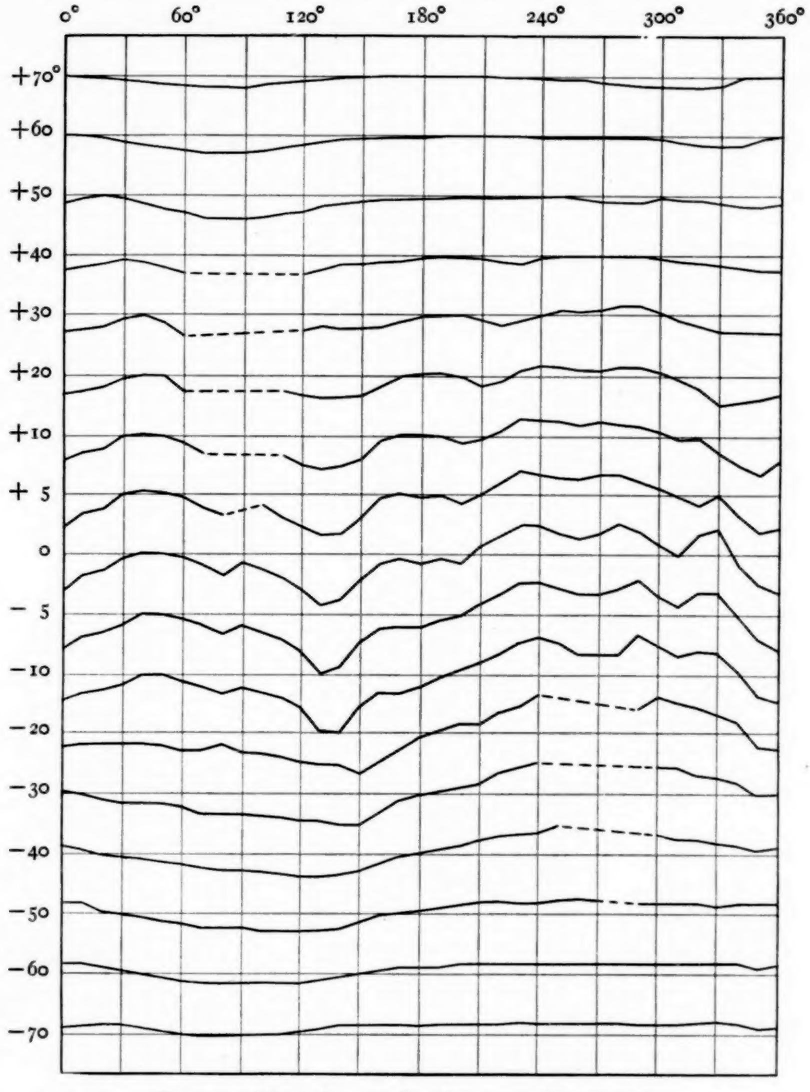

FIG. 10.—Adopted corrections for $m = 11$ (See Fig. 9)

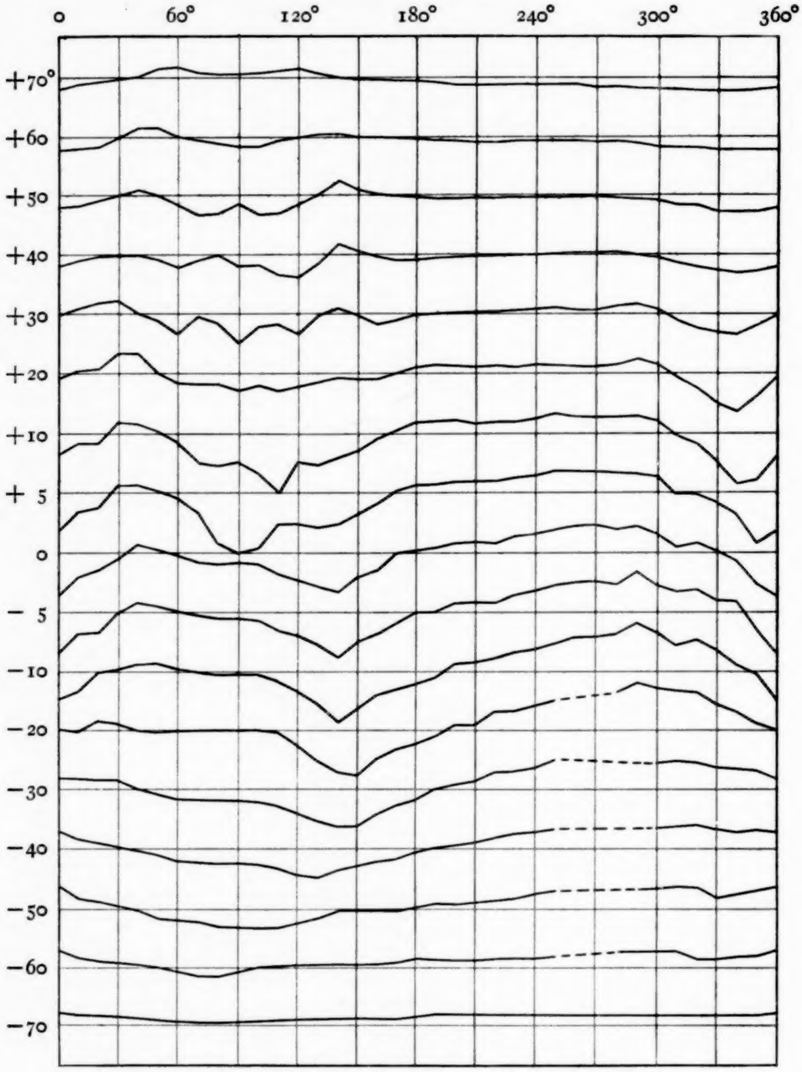

FIG. 11.—Adopted corrections for $m=13.5$ (See Fig. 9)

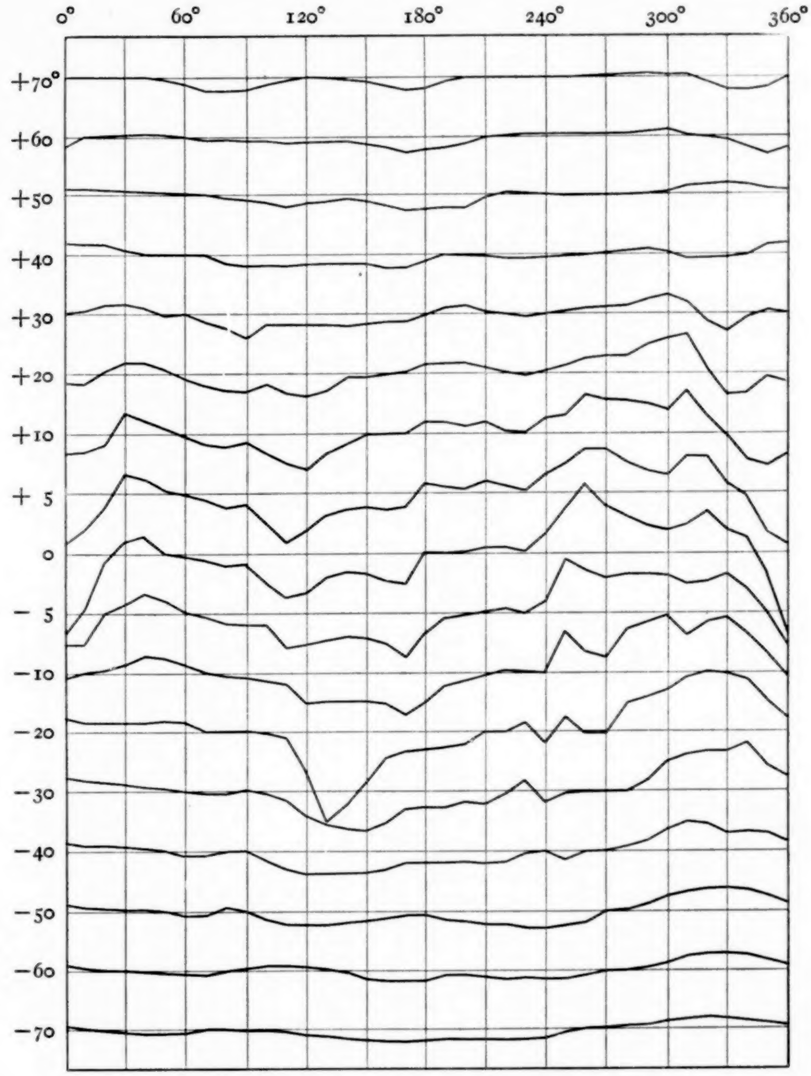

FIG. 12.—Adopted corrections for $m=16$ (See Fig. 9)

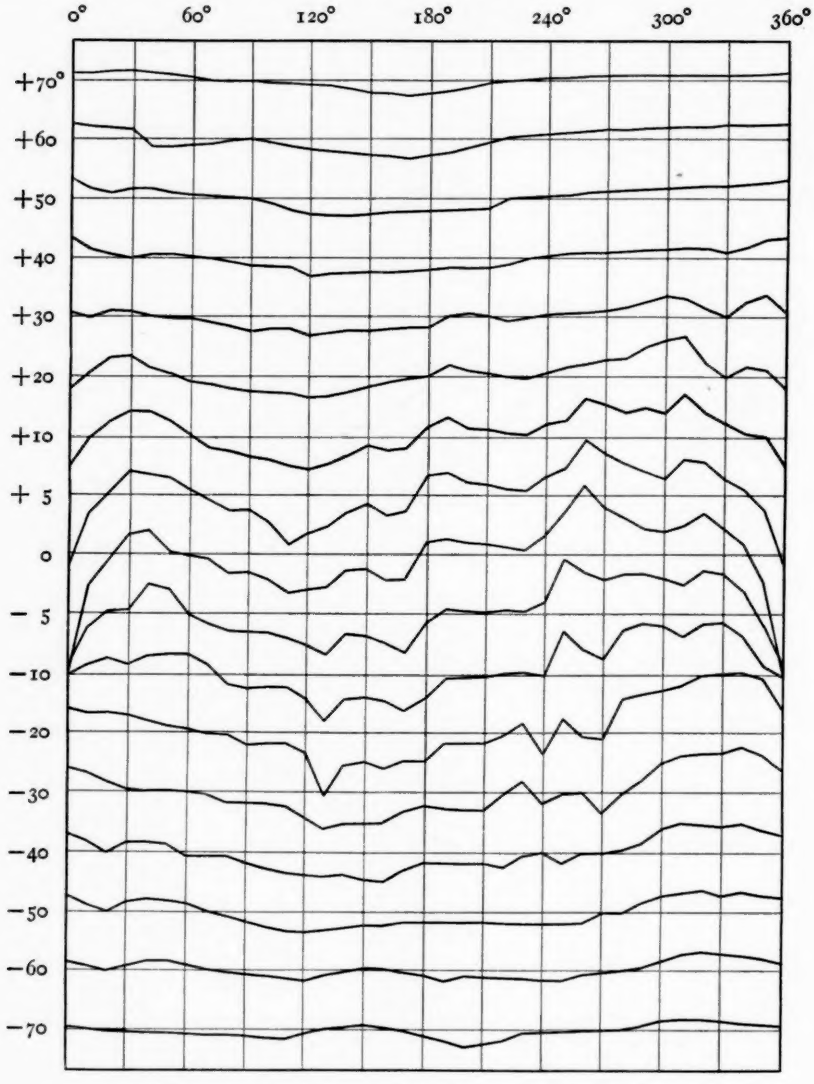

FIG. 13.—Adopted corrections for $m=18$ (See Fig. 9)

TABLE XIV

Mean Distribution of Stars*

m	0°	5°	10°	15°	20°	25°	30°
4.0	8.193_{230}	8.168_{230}	8.117_{230}	8.054_{230}	7.989_{230}	7.930_{230}	7.870_{231}
4.5	8.423_{229}	8.398_{229}	8.347_{229}	8.284_{229}	8.219_{229}	8.160_{229}	8.101_{230}
5.0	8.652_{228}	8.627_{228}	8.576_{228}	8.513_{228}	8.448_{228}	8.389_{228}	8.331_{229}
5.5	8.880_{227}	8.855_{227}	8.804_{226}	8.741_{226}	8.676_{226}	8.617_{227}	8.560_{228}
6.0	9.107_{226}	9.082_{226}	9.030_{225}	8.967_{225}	8.902_{225}	8.844_{226}	8.788_{226}
6.5	9.333_{225}	9.308_{225}	9.255_{224}	9.192_{224}	9.127_{223}	9.070_{224}	9.014_{225}
7.0	9.558_{224}	9.533_{223}	9.479_{222}	9.416_{222}	9.350_{221}	9.294_{222}	9.239_{223}
7.5	9.782_{223}	9.756_{222}	9.701_{220}	9.638_{219}	9.571_{219}	9.516_{220}	9.462_{221}
8.0	0.005_{222}	9.978_{220}	9.921_{219}	9.857_{218}	9.790_{217}	9.736_{217}	9.683_{218}
8.5	0.227_{221}	0.198_{219}	0.140_{217}	0.075_{216}	0.007_{215}	9.953_{215}	9.901_{216}
9.0	0.448_{220}	0.417_{218}	0.357_{216}	0.291_{214}	0.222_{213}	0.168_{213}	0.117_{214}
9.5	0.668_{219}	0.635_{217}	0.573_{215}	0.505_{212}	0.435_{211}	0.381_{211}	0.331_{212}
10.0	0.887_{217}	0.852_{215}	0.788_{213}	0.717_{210}	0.646_{209}	0.592_{209}	0.543_{209}
10.5	1.104_{215}	1.067_{213}	1.001_{211}	0.927_{208}	0.855_{208}	0.801_{206}	0.752_{205}
11.0	1.319_{214}	1.280_{212}	1.212_{209}	1.135_{206}	1.063_{206}	1.007_{203}	0.957_{201}
11.5	1.533_{212}	1.492_{210}	1.421_{207}	1.341_{204}	1.269_{204}	1.210_{200}	1.158_{197}
12.0	1.745_{210}	1.702_{208}	1.628_{205}	1.545_{202}	1.473_{200}	1.410_{197}	1.355_{193}
12.5	1.955_{208}	1.910_{206}	1.833_{202}	1.747_{199}	1.673_{195}	1.607_{192}	1.548_{188}
13.0	2.163_{205}	2.116_{203}	2.035_{200}	1.946_{196}	1.868_{191}	1.799_{187}	1.736_{183}
13.5	2.368_{201}	2.319_{200}	2.235_{198}	2.142_{193}	2.059_{186}	1.986_{181}	1.919_{178}
14.0	2.569_{197}	2.519_{197}	2.433_{195}	2.335_{190}	2.245_{181}	2.167_{175}	2.097_{172}
14.5	2.766_{193}	2.716_{194}	2.628_{191}	2.525_{186}	2.426_{176}	2.342_{170}	2.269_{166}
15.0	2.959_{188}	2.910_{190}	2.819_{188}	2.711_{182}	2.602_{172}	2.512_{165}	2.435_{160}
15.5	3.147_{183}	3.100_{185}	3.007_{184}	2.893_{177}	2.774_{167}	2.677_{160}	2.595_{154}
16.0	3.330_{177}	3.285_{180}	3.191_{179}	3.070_{172}	2.941_{162}	2.837_{154}	2.749_{147}
16.5	3.507_{172}	3.465_{174}	3.370_{174}	3.242_{167}	3.103_{157}	2.991_{148}	2.896_{140}
17.0	3.679_{167}	3.639_{168}	3.544_{168}	3.409_{162}	3.260_{152}	3.139_{142}	3.036_{134}
17.5	3.846_{162}	3.807_{162}	3.712_{162}	3.571_{156}	3.412_{147}	3.281_{136}	3.170_{128}
18.0	4.008_{157}	3.969_{156}	3.874_{156}	3.727_{150}	3.559_{142}	3.417_{130}	3.298_{122}
18.5	4.165_{152}	4.125_{150}	4.030_{150}	3.877_{144}	3.701_{137}	3.547_{124}	3.420_{116}
19.0	4.317_{146}	4.275_{144}	4.180_{144}	4.021_{138}	3.838_{131}	3.671_{118}	3.536_{110}
19.5	4.463_{140}	4.419_{138}	4.324_{138}	4.159_{132}	3.969_{125}	3.789_{112}	3.646_{104}
20.0	4.603_{135}	4.557_{132}	4.462_{131}	4.291_{126}	4.094_{119}	3.901_{106}	3.750_{98}
20.5	4.738_{129}	4.689_{126}	4.593_{124}	4.417_{120}	4.213_{113}	4.007_{100}	3.848_{91}
21.0	4.867	4.815	4.717	4.537	4.326	4.107	3.939

* The table gives $\log N_m$; N_m=number of stars per square degree brighter than m, international photographic scale. Pole of Milky Way: $\alpha=12^h41^m20^s$, $\delta=27°21'$ (1875).

TABLE XIV—*Continued*

m	30°	40°	50°	60°	70°	80°	90°
4.0	7.870 231	7.784 230	7.744 229	7.711 229	7.689 231	7.668 230	7.655 230
4.5	8.101 230	8.014 229	7.973 228	7.940 229	7.920 230	7.898 230	7.885 230
5.0	8.331 229	8.243 228	8.201 227	8.169 228	8.150 228	8.128 228	8.115 229
5.5	8.560 228	8.471 227	8.428 226	8.397 227	8.378 226	8.356 225	8.344 226
6.0	8.788 226	8.698 226	8.654 225	8.624 225	8.604 224	8.581 223	8.570 223
6.5	9.014 225	8.924 225	8.879 224	8.849 223	8.828 221	8.804 220	8.793 220
7.0	9.239 223	9.149 223	9.103 222	9.072 221	9.049 218	9.024 218	9.013 217
7.5	9.462 221	9.372 221	9.325 220	9.293 219	9.267 215	9.242 215	9.230 214
8.0	9.683 218	9.593 219	9.545 218	9.512 216	9.482 212	9.457 212	9.444 210
8.5	9.901 216	9.812 217	9.763 215	9.728 212	9.694 208	9.669 207	9.654 205
9.0	0.117 214	0.029 214	9.978 211	9.940 207	9.902 204	9.876 202	9.859 201
9.5	0.331 212	0.243 210	0.189 206	0.147 202	0.106 199	0.078 197	0.060 197
10.0	0.543 209	0.453 206	0.395 202	0.349 197	0.305 195	0.275 192	0.257 192
10.5	0.752 205	0.659 202	0.597 198	0.546 192	0.500 190	0.467 187	0.449 187
11.0	0.957 201	0.861 198	0.795 194	0.738 187	0.690 184	0.654 182	0.636 182
11.5	1.158 197	1.059 193	0.989 189	0.925 182	0.874 179	0.836 177	0.818 177
12.0	1.355 193	1.252 188	1.178 183	1.107 177	1.053 173	1.013 172	0.995 171
12.5	1.548 188	1.440 183	1.361 177	1.284 172	1.226 168	1.185 167	1.166 165
13.0	1.736 183	1.623 178	1.538 172	1.456 167	1.394 163	1.352 161	1.331 160
13.5	1.919 178	1.801 172	1.710 166	1.623 162	1.557 158	1.513 156	1.491 155
14.0	2.097 172	1.973 166	1.876 160	1.785 156	1.715 152	1.669 150	1.646 150
14.5	2.269 166	2.139 160	2.036 153	1.941 150	1.867 146	1.819 144	1.796 144
15.0	2.435 160	2.299 153	2.189 146	2.091 144	2.013 140	1.963 138	1.940 138
15.5	2.595 154	2.452 146	2.335 140	2.235 138	2.153 134	2.101 132	2.078 133
16.0	2.749 147	2.598 140	2.475 134	2.373 132	2.287 128	2.233 127	2.211 127
16.5	2.896 140	2.738 133	2.609 128	2.505 126	2.415 123	2.360 122	2.338 121
17.0	3.036 134	2.871 127	2.737 122	2.631 120	2.538 118	2.482 117	2.459 115
17.5	3.170 128	2.998 120	2.859 116	2.751 114	2.656 113	2.599 111	2.574 109
18.0	3.298 122	3.118 113	2.975 110	2.865 108	2.769 107	2.710 105	2.683 104
18.5	3.420 116	3.231 107	3.085 104	2.973 102	2.876 102	2.815 100	2.787 99
19.0	3.536 110	3.338 100	3.189 98	3.075 96	2.978 96	2.915 94	2.886 93
19.5	3.646 104	3.438 94	3.287 92	3.171 90	3.074 90	3.009 88	2.979 87
20.0	3.750 98	3.532 87	3.379 86	3.261 84	3.164 84	3.097 82	3.066 81
20.5	3.848 91	3.619 81	3.465 80	3.345 78	3.248 78	3.179 76	3.147 75
21.0	3.939	3.700	3.545	3.423	3.326	3.255	3.222

Tables XV*a–e* give the data in their final form, which is that of a correction to be added to the values of $\log N_m$ in Table XIV in order to pass from the mean to the actual distribution at any point in the sky. The "actual" distribution thus found is, of course, only an approximation to the true distribution, which for various reasons cannot be perfectly represented by the present results. The large fields of the Astrographic Zones, the small, widely spaced fields of the Selected Areas, and the smoothing introduced by the use of mean values and curves of equal density all tend to obscure local irregularities of density. For the present, however, we are interested chiefly in the larger features of the distribution, and these are well shown by the data. In this respect the sampling of the faint stars provided by the Selected Areas is quite satisfactory. Serious uncertainty arises only in the case of heavily obscured regions like those in Taurus and Ophiuchus. On this account no attempt has been made to indicate the contour lines in the deep minimum surrounding S.A. 110 at $\alpha = 18^h36^m$, $\delta = 0^\circ$ (see Fig. 8).

The results for $m = 9$ and 11, which depend entirely on the thirty-nine Astrographic Zones summarized in Table XI, are the least reliable. The declinations between $+29^\circ$ and -42° are well covered by these zones, but the only others for which data are available are $+62^\circ$ and -65°. The equal-density curves of Figures 4 and 5 are therefore very uncertain outside the intermediate latitudes. The corresponding deviations in Tables XV, which are inclosed in parentheses, are interpolated values and are represented by dotted intervals in the curves of Figures 9 and 10. Further, it has been assumed that the mean zonal density for these limits deviates systematically from the average distribution in the same way and to the same extent as for the fainter stars, which, as explained above, is supported only by the circumstance that the change in the systematic deviation with magnitude seems to be slow.

Aside from the well-known obscured regions and the very high density in certain portions of the Milky Way, the most significant feature is the general asymmetry of distribution revealed by the fact that practically all the deviations in certain longitudes are positive, while those in diametrically opposite longitudes are negative. The phenomenon appears in the distribution to all magnitude limits and

TABLE XV*a*

CORRECTIONS TO MEAN DISTRIBUTION, TABLE XIV
(Unit = 0.01 in log N_m)

$m = 9.0$

Long.	Galactic Latitude														
	+70°	+60°	+50°	+40°	+30°	+20°	+10°	0°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°	0	− 2	−10	−10	− 9	−10	−18	−26	−20	−12	− 1	+ 3	+ 6	+ 7	+10
10	0	− 5	− 7	−20	−10	−10	− 8	−20	−20	−10	− 3	+ 3	+ 6	+10	+12
20	− 1	− 6	(− 8)	− 8	− 8	−10	−11	−15	−15	− 9	− 5	0	+ 3	+ 9	+14
30	− 2	− 7	(−10)	(−10)	(−12)	(−14)	− 8	− 9	− 7	− 7	− 8	− 4	− 2	+ 4	+13
40	− 3	(−10)	(−14)	(−14)	(−16)	(−20)	(− 6)	(− 8)	− 7	−12	−13	− 6	− 5	− 1	+10
50	− 4	(−12)	(−16)	(−18)	(−20)	(−26)	(− 2)	(− 6)	(− 8)	−14	−16	−10	− 6	− 3	+ 5
60	− 5	(−16)	−20	−20	−25	−30	0	(− 4)	(−12)	(−14)	−12	−10	− 9	− 4	0
70	− 6	−20	−25	(−20)	(−22)	(−28)	+ 6	0	(−16)	(−14)	−12	−10	−10	− 5	0
80	− 6	−25	(−20)	(−20)	(−22)	(−26)	(+ 2)	0	(−18)	(−14)	−12	−12	−12	− 6	− 1
90	− 6	−30	(−15)	(−22)	(−22)	(−22)	(− 4)	(− 6)	(−22)	(−14)	− 8	−16	−13	− 8	− 1
100	− 5	−20	−10	(−24)	(−21)	(−20)	(− 8)	(−10)	(−28)	(−15)	− 8	−20	−15	−11	0
110	− 5	−20	−20	(−28)	(−20)	(−18)	−12	(−14)	(−32)	(−15)	− 8	−20	−16	−12	+ 1
120	− 4	(−16)	−20	−30	−20	−15	−12	(−20)	(−36)	−15	−10	−22	−16	−12	+ 5
130	− 3	(−12)	(−18)	(−28)	(−20)	(−16)	(−16)	(−26)	−40	−22	−22	−22	−16	−11	+ 8
140	− 2	(− 8)	(−15)	(−24)	(−20)	(−18)	(−20)	−30	−40	−30	−30	−22	−15	− 6	+11
150	− 1	− 5	(−12)	(−20)	(−20)	(−20)	−20	−25	−28	−28	−26	−18	−12	+ 1	+15
160	− 1	− 6	−10	−15	−20	−20	−10	−15	−21	−14	−10	− 7	− 4	+ 8	+15
170	0	− 5	− 8	−12	−14	0	+ 3	−10	−12	− 6	− 2	0	+ 3	+11	+14
180	0	− 3	− 3	− 6	−10	− 2	− 5	−11	− 5	+ 2	+ 8	+ 9	+ 7	+12	+12
190	0	0	0	0	− 5	− 7	−10	−10	+ 2	+10	+10	+ 9	+12	+12	+10
200	0	0	− 3	− 5	− 7	−10	−12	− 6	+12	+19	+12	+ 8	+10	+10	+14
210	0	0	− 4	− 8	−10	−12	− 6	+10	+24	+20	+14	+12	+12	+11	+15
220	+ 1	0	− 4	− 9	−11	− 6	+ 6	+22	+28	+26	+24	+18	+15	+10	+16
230	0	+ 1	− 3	−10	− 8	+ 3	+22	+40	+40	+33	+28	+22	+18	+14	+16
240	0	+ 2	0	−10	− 2	+12	+16	+20	+30	+36	+33	+26	+18	+16	+16
250	− 2	+ 2	− 1	− 5	+10	+10	+12	(+22)	+40	(+34)	(+32)	+28	+20	+18	+17
260	− 3	+ 1	0	− 2	+ 8	+10	(+10)	(+26)	(+40)	(+32)	(+30)	(+26)	+25	+18	+18
270	− 4	0	0	0	0	−13	(+ 8)	(+28)	(+40)	(+30)	(+28)	(+24)	+20	+18	+18
280	− 6	− 2	0	0	0	+ 6	(+ 4)	+30	(+40)	(+26)	(+26)	(+22)	+15	+15	+15
290	− 9	− 4	− 3	− 5	0	0	0	(+20)	+40	(+22)	(+22)	(+20)	+15	+16	+18
300	−11	− 6	− 5	− 9	0	− 2	0	(+10)	(+30)	(+18)	+20	+20	(+14)	+17	+20
310	−12	− 9	− 7	− 8	− 5	+ 2	− 2	0	+15	(+16)	(+15)	(+15)	(+12)	+15	+20
320	−11	− 9	− 7	− 8	− 7	− 2	+ 6	+20	+20	+15	+10	+10	+10	+15	+15
330	− 9	−10	− 8	− 8	−10	−19	−10	+10	+29	+25	+12	+ 8	+ 8	+10	+12
340	0	−10	−10	− 9	−14	−20	−24	−14	+ 3	+ 7	+ 6	+ 5	+ 6	+ 8	+ 9
350	0	− 5	−10	−10	−13	−19	−30	−25	−24	−10	0	+ 3	+ 5	+ 7	+ 9

$m = 11.0$

TABLE XV*b*

CORRECTIONS TO MEAN DISTRIBUTION, TABLE XIV
(Unit=0.01 in log N_m)

LONG.	GALACTIC LATITUDE																
	+70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0°	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°....	0	0	−8	−14	−16	−18	−26	−33	−36	−33	−26	−13	+2	+8	+10	+10	+8
10....	−1	0	−4	−11	−14	−15	−17	−20	−24	−24	−20	−10	−2	+5	+10	+10	+9
20....	−3	−3	0	−8	−11	−11	−13	−15	−16	−18	−15	−10	−6	−2	+2	+6	+10
30....	−5	−6	−4	−5	−5	−3	0	0	−5	−10	−10	−10	−8	−4	−3	+2	+10
40....	−6	−9	−9	−7	−2	0	+2	+3	+2	+1	0	−10	−8	−6	−5	−2	+7
50....	−8	−12	−14	−13	−8	0	0	+1	0	0	0	−12	−9	−8	−8	−4	+4
60....	−9	−14	−17	−19	−22	−14	−6	−4	−4	−5	−7	−16	−13	−10	−11	−6	0
70....	−10	−17	−23	(−20)	(−22)	(−16)	−18	−14	−10	−10	−12	−16	−20	−14	−13	−7	−1
80....	−10	−17	−24	(−20)	(−20)	(−16)	(−19)	−22	−22	−20	−20	−10	−20	−17	−13	−8	−1
90....	−11	−17	−24	(−20)	(−19)	(−16)	(−20)	(−16)	−8	−10	−13	−18	−20	−17	−13	−8	−1
100....	−8	−15	−22	(−20)	(−18)	(−15)	(−20)	−10	−15	−18	−18	−20	−23	−19	−16	−8	−1
110....	−7	−13	−18	(−20)	(−16)	−15	−20	−25	−25	−25	−24	−24	−24	−22	−17	−9	0
120....	−5	−9	−16	−20	−15	−20	−30	−33	−35	−36	−34	−28	−26	−24	−17	−9	+3
130....	−4	−7	−10	−14	−13	−23	−34	−42	−51	−58	−58	−30	−26	−24	−16	−7	+6
140....	−2	−5	−8	−9	−14	−22	−31	−40	−46	−52	−60	−30	−30	−21	−14	−4	+10
150....	−1	−4	−7	−9	−14	−20	−25	−25	−26	−27	−32	−40	−30	−17	−9	0	+10
160....	0	−3	−5	−8	−12	−10	−5	−5	−10	−14	−19	−28	−18	−9	−4	+4	+10
170....	0	−2	−5	−7	−6	0	+1	0	−5	−13	−20	−16	−7	−4	0	+6	+10
180....	0	−2	−4	−3	−3	+3	+1	−3	−9	−13	−12	−5	−2	+1	+3	+6	+9
190....	0	0	−3	−2	0	+4	0	−2	−5	−5	−4	+2	+4	+5	+6	+7	+10
200....	0	0	−2	−2	0	0	−6	−8	−9	−2	+5	+9	+6	+8	+9	+10	+10
210....	0	0	−2	−3	−5	−9	−4	0	+9	+12	+12	+9	+10	+14	+12	+10	+10
220....	−1	0	−2	−6	−10	−5	+6	+12	+18	+19	+20	+21	+23	+18	+12	+10	+10
230....	−1	0	−2	−7	−6	+6	+18	+25	+29	+32	+33	+28	+27	+20	+11	+10	+11
240....	−2	−1	0	−2	0	+12	+17	+20	+28	+33	+36	+40	+32	+22	+11	+10	+11
250....	−3	−2	0	0	+5	+10	+15	+17	+20	+26	+32	(+36)	(+32)	+30	+13	+10	+12
260....	−4	−2	−3	0	+4	+7	+13	+15	+15	+20	+20	(+34)	(+32)	(+28)	+14	+10	+12
270....	−6	−2	−5	0	+6	+7	+15	+20	+20	+20	+20	(+32)	(+30)	(+26)	+13	+10	+12
280....	−7	−2	−5	0	+10	+10	+12	+20	+30	+26	+20	(+28)	(+30)	(+24)	(+12)	+10	+12
290....	−8	−2	−5	0	+10	+10	+10	+14	+24	+35	+40	+26	(+28)	(+22)	+10	+10	+10
300....	−10	−3	−2	−3	+3	+5	+5	+6	+10	+18	+30	+37	+28	+20	+10	+10	+10
310....	−10	−6	−4	−5	−5	−4	−4	−2	0	+8	+18	+33	+28	+15	+10	+10	+10
320....	−10	−8	−5	−6	−10	−12	−2	−10	+20	+21	+25	+27	+19	+15	+10	+10	+11
330....	−8	−9	−7	−8	−15	−28	−16	0	+26	+21	+23	+20	+16	+12	+7	+10	+12
340....	0	−9	−10	−11	−17	−26	−30	−22	−10	−3	+5	+12	+12	+10	+10	+10	+11
350....	0	−4	−10	−14	−16	−23	−40	−38	−31	−25	−22	−12	0	+5	+10	+7	+6

TABLE XVc

CORRECTIONS TO MEAN DISTRIBUTION, TABLE XIV
(Unit = 0.01 in log N_m)

$m = 13.5$

Long.	Galactic Latitude																
	+70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0°	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°....	−10	−12	−11	−10	− 1	− 5	−21	−36	−44	−40	−27	+ 1	+13	+18	+20	+16	+12
10....	− 7	−11	−10	− 5	+ 6	+ 4	− 9	−18	−25	−22	−20	− 1	+12	+10	+ 9	+10	+11
20....	− 5	−10	− 5	− 2	+13	+ 5	− 9	−14	−16	−21	− 1	+10	+10	+ 7	+ 6	+ 7	+10
30....	− 2	0	− 1	− 2	+14	+20	+12	+ 8	− 6	− 1	+ 2	+ 7	+10	+ 3	+ 2	+ 5	+ 8
40....	+ 1	+10	+ 5	− 1	0	+20	+10	+ 8	+ 9	+10	+ 8	− 1	0	− 2	− 3	+ 3	+ 7
50....	+ 8	+10	− 1	− 5	− 6	0	+ 3	+ 2	+ 4	+ 6	+ 9	− 2	− 5	− 6	− 9	0	+ 5
60....	+10	0	−10	−12	−20	− 9	− 8	− 5	− 3	+ 1	+ 4	0	− 8	−11	−12	− 5	+ 4
70....	+ 5	− 4	−20	− 5	− 4	−10	−30	−19	− 8	− 4	0	0	−10	−12	−13	− 8	+ 3
80....	+ 4	− 7	−18	0	− 8	−10	−32	−50	−10	− 6	− 4	0	−10	−14	−18	− 8	+ 3
90....	+ 4	− 8	− 8	−11	−28	−17	−28	−60	− 9	− 6	− 3	0	−10	−14	−19	− 5	+ 4
100....	+ 5	− 8	−20	−10	−12	−12	−40	−55	−10	− 7	− 4	0	−11	−15	−20	0	+ 4
110....	+ 7	− 4	−18	−20	−10	−18	−60	−30	−21	−18	− 8	− 3	−15	−19	−20	+ 1	+ 5
120....	+10	0	− 8	−23	−20	−12	−29	−30	−27	−24	−20	−15	−24	−26	−14	+ 2	+ 6
130....	+ 5	+ 4	0	− 4	− 3	− 8	−31	−34	−33	−33	−33	−32	−32	−28	− 9	+ 3	+ 6
140....	0	+ 4	+14	+10	+ 6	− 5	−25	−30	−40	−45	−50	−44	−37	−21	− 4	+ 3	+ 6
150....	− 1	0	+ 5	+ 4	− 2	− 6	−16	−22	−25	−29	−37	−46	−36	−16	− 3	+ 3	+ 7
160....	− 2	0	0	− 3	− 9	− 6	− 6	−10	−16	−22	−24	−28	−25	−13	− 4	+ 3	+ 7
170....	− 3	− 1	− 1	− 5	− 6	0	+ 4	+ 3	0	−11	−16	−19	−15	−10	− 4	+ 5	+ 7
180....	− 4	− 2	− 2	− 5	− 1	+ 6	+11	+ 8	+ 4	0	−12	−14	−10	− 4	+ 1	+10	+ 9
190....	− 5	− 3	− 3	− 3	0	+ 8	+12	+ 9	+ 5	0	− 6	− 6	0	+ 2	+ 6	+ 9	+12
200....	− 6	− 4	− 4	− 2	+ 1	+ 7	+13	+12	+10	+ 9	+ 9	+ 6	+ 5	+ 4	+ 5	+ 8	+11
210....	− 6	− 5	− 3	− 1	+ 2	+ 6	+11	+12	+11	+10	+10	+ 6	+ 9	+ 7	+ 6	+ 8	+10
220....	− 6	− 5	− 3	− 1	+ 3	+ 8	+12	+12	+10	+10	+14	+20	+17	+12	+ 8	+ 9	+10
230....	− 6	− 4	− 2	0	+ 4	+ 7	+12	+15	+16	+18	+21	+20	+17	+15	+10	+10	+10
240....	− 6	− 4	− 2	0	+ 5	+10	+15	+17	+19	+22	+24	+26	+22	+17	+14	+10	+10
250....	− 6	− 4	− 2	+ 2	+ 6	+ 9	+20	+23	+25	+27	+30	+33	+32	+20	+16	+12	+10
260....	− 6	− 4	− 1	+ 3	+ 5	+ 8	+16	+22	+27	+31	+35	(+35)	(+32)	(+20)	(+17)	(+13)	+10
270....	− 8	− 5	− 1	+ 2	+ 5	+ 8	+16	+22	+28	+32	+35	(+35)	(+32)	(+20)	(+17)	(+14)	+10
280....	− 8	− 5	− 2	+ 4	+10	+10	+16	+20	+25	+29	+38	+40	(+30)	(+20)	(+17)	+15	+10
290....	− 9	− 6	− 4	0	+12	+15	+18	+20	+27	+43	+50	+50	(+29)	(+22)	+18	+15	+10
300....	−10	− 8	− 5	− 2	+ 6	+11	+13	+16	+20	+28	+42	+45	+28	+23	+18	+15	+10
310....	−10	− 9	− 8	− 7	− 6	− 4	− 3	− 2	+ 7	+21	+28	+42	+30	+24	+21	+15	+10
320....	−11	−10	−10	−11	−13	−14	−10	− 2	+11	+25	+34	+40	+29	+25	+19	+ 9	+10
330....	−12	−12	−15	−14	−17	−30	−28	− 8	+ 4	+13	+25	+28	+25	+20	+10	+ 9	+10
340....	−12	−12	−15	−16	−20	−36	−50	−20	− 6	+12	+ 8	+20	+22	+17	+14	+12	+10
350....	−11	−12	−15	−14	−11	−24	−46	−50	−30	−15	0	+ 8	+20	+20	+17	+12	+10

TABLE XV*d*

CORRECTIONS TO MEAN DISTRIBUTION, TABLE XIV
(Unit = 0.01 in log N_m)

$m = 16.0$

Long.	Galactic Latitude +70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0°	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°	0	−10	+6	+12	+2	−8	−20	−50	−80	−30	−5	+14	+14	+10	+8	+6	+3
10	0	0	+6	+11	+4	−10	−18	−35	−55	−30	0	+10	+11	+7	+5	+3	0
20	0	+1	+5	+10	+10	+4	−10	−13	−8	0	+4	+10	+10	+7	+4	+1	−2
30	0	+2	+4	+5	+11	+12	+22	+20	+12	+9	+9	+10	+7	+6	+3	0	−4
40	0	+3	+3	+1	+6	+12	+14	+14	+16	+20	+17	+10	+5	+4	+2	−2	−5
50	−3	+2	+2	0	−3	+5	+6	+3	+1	+12	+15	+12	+3	+2	0	−3	−5
60	−7	0	+1	0	0	−6	−3	−3	−4	+3	+9	+10	0	−5	−5	−4	−5
70	−12	−4	0	0	−8	−12	−10	−7	−7	−4	0	0	−3	−5	−5	−5	0
80	−12	−4	−4	−8	−14	−17	−13	−13	−11	−9	−4	0	−3	0	+5	0	0
90	−11	−5	−6	−11	−25	−18	−8	−10	−10	−10	−5	+2	+2	+1	0	+3	−2
100	−7	−5	−8	−10	−10	−10	−20	−30	−29	−10	−6	−2	−2	−9	−8	+5	−2
110	−4	−7	−12	−10	−10	−20	−30	−49	−45	−33	−9	−7	−8	−18	−12	+5	−3
120	0	−6	−8	−9	−10	−24	−35	−36	−40	−30	−30	−40	−25	−23	−12	+4	−6
130	0	−6	−7	−8	−10	−17	−20	−23	−24	−26	−28	−90	−33	−23	−12	+1	−7
140	−2	−6	−5	−8	−9	−5	−10	−15	−19	−24	−28	−72	−36	−23	−10	−3	−9
150	−5	−8	−7	−9	−10	−5	−2	−12	−20	−25	−28	−51	−40	−22	−9	−8	−11
160	−9	−11	−11	−13	−8	−2	0	−15	−27	−30	−30	−26	−31	−18	−7	−9	−12
170	−12	−15	−15	−13	−8	+2	0	−12	−30	−44	−43	−20	−16	−12	−5	−9	−12
180	−11	−12	−14	−7	−2	+9	+12	+11	+2	−20	−30	−18	−15	−12	−5	−10	−11
190	−5	−10	−12	0	+6	+11	+12	+7	+1	−6	−12	−16	−15	−12	−8	−5	−10
200	0	−7	−12	−1	+8	+11	+8	+5	+2	−3	−8	−12	−10	−10	−10	−5	−10
210	0	0	−3	−2	+4	+6	+12	+12	+7	+1	−3	0	−12	−12	−12	−7	−10
220	0	+2	+3	−4	0	+1	+4	+8	+7	+5	+3	0	−4	−11	−13	−8	−10
230	0	+4	+2	−4	−4	−2	+2	+3	+4	0	+1	+11	+13	−4	−15	−7	−10
240	0	+4	0	−3	0	+4	+14	+18	+19	+12	0	−10	−10	0	−15	−8	−9
250	0	+4	−1	−2	+4	+9	+19	+30	+46	+55	+43	+15	−2	−8	−12	−8	−4
260	+1	+4	−1	−1	+6	+14	+38	+45	+70	+45	+23	−2	0	0	−10	−5	0
270	+2	+4	0	+2	+7	+16	+33	+45	+49	+35	+16	−2	0	0	0	−1	+1
280	+4	+4	0	+4	+9	+17	+32	+31	+36	+40	+45	+30	0	+5	+2	0	+2
290	+5	+6	+1	+5	+15	+30	+30	+24	+28	+40	+52	+35	+11	+10	+8	+4	+3
300	+3	+8	+4	+3	+20	+34	+25	+18	+25	+39	+56	+44	+30	+23	+14	+9	+8
310	+2	+2	+8	−5	+13	+40	+42	+36	+30	+31	+38	+55	+36	+30	+19	+14	+10
320	−5	0	+11	−4	−8	+5	+16	+36	+43	+33	+52	+61	+40	+29	+22	+16	+11
330	−11	−3	+12	−4	−16	−23	−4	+10	+25	+40	+55	+60	+40	+18	+23	+16	+10
340	−11	−10	+11	0	−5	−20	−26	−3	+15	+25	+40	+54	+49	+20	+20	+14	+8
350	−8	−17	+7	+10	+4	−4	−32	−38	−20	0	+20	+33	+26	+19	+14	+10	+6

TABLE XVe

CORRECTIONS TO MEAN DISTRIBUTION, TABLE XIV
(Unit = 0.01 in log N_m)

$m = 18.0$

Long.	Galactic Latitude																
	+70°	+60°	+50°	+40°	+30°	+20°	+10°	+5°	0	−5°	−10°	−20°	−30°	−40°	−50°	−60°	−70°
0°....	+9	+15	+20	+21	+5	−11	−30	−70	−118	−50	0	+26	+25	+18	+14	+9	+6
10....	+9	+12	+10	+8	−1	+6	−1	−18	−30	−12	+10	+20	+20	+12	+8	+6	+2
20....	+10	+11	+7	+4	+7	+20	+16	+4	−7	+4	+16	+22	+10	0	0	0	−1
30....	+10	+10	+10	−1	+6	+21	+26	+27	+19	+5	+10	+18	+4	+10	+10	+6	−2
40....	+9	−8	+10	+4	0	+9	+26	+20	+25	+30	+18	+12	0	+10	+13	+10	−3
50....	+7	−8	+7	+4	−3	+4	+15	+16	+3	+25	+20	+6	+1	+8	+11	+10	−4
60....	+4	−7	+4	+1	−3	−6	+4	+6	−2	0	+20	+4	0	−5	+8	+5	−5
70....	0	−6	+3	−1	−6	−7	−10	−5	−5	−9	+10	−2	−4	−5	0	0	−6
80....	−1	−2	+1	−5	−10	−12	−14	−16	−20	−16	−10	−3	−10	−5	−5	−4	−6
90....	−1	0	−2	−8	−15	−15	−20	−15	−18	−16	−13	−12	−10	−10	−10	−5	−7
100....	−3	−4	−6	−8	−12	−15	−24	−29	−27	−18	−12	−11	−10	−16	−16	−6	−8
110....	−4	−7	−11	−10	−12	−16	−30	−50	−40	−23	−12	−10	−14	−22	−20	−8	−9
120....	−5	−10	−15	−20	−20	−22	−34	−40	−36	−30	−26	−20	−26	−24	−21	−10	−5
130....	−6	−12	−16	−17	−17	−20	−29	−32	−34	−40	−48	−65	−36	−25	−20	−6	0
140....	−9	−13	−17	−16	−14	−15	−20	−18	−17	−20	−25	−33	−30	−24	−16	−2	+2
150....	−12	−15	−15	−15	−14	−10	−8	−9	−15	−22	−24	−29	−30	−28	−14	+2	+5
160....	−13	−17	−14	−15	−12	−7	−14	−22	−27	−28	−27	−35	−29	−29	−14	+1	+2
170....	−15	−20	−13	−14	−10	−4	−12	−16	−26	−37	−37	−28	−18	−18	−10	−3	−2
180....	−12	−15	−12	−13	−10	0	+10	+20	+12	−9	−25	−28	−13	−10	−10	−7	−6
190....	−10	−12	−11	−10	0	+11	+20	+23	+15	+5	−4	−10	−15	−10	−10	−10	−10
200....	−6	−8	−10	−10	+2	+6	+9	+12	+11	+3	−4	−10	−16	−10	−10	−6	−16
210....	−3	−4	−10	−10	0	+4	+8	+11	+10	+2	−3	−10	−17	−10	−10	−7	−14
220....	0	+2	0	−7	−5	0	+4	+7	+7	+5	+3	−4	−4	−13	−11	−7	−10
230....	+2	+4	+2	−2	−3	−2	+2	+5	+5	+4	+3	+10	+12	−5	−12	−8	−4
240....	+3	+5	+3	+1	+2	+4	+14	+18	+18	+12	0	−20	−10	0	−12	−9	−3
250....	+4	+6	+4	+3	+4	+10	+18	+28	+45	+55	+44	+14	−2	−10	−11	−9	−2
260....	+6	+8	+6	+5	+4	+14	+36	+56	+70	+43	+25	−5	0	0	−10	−6	−1
270....	+6	+10	+7	+5	+6	+16	+33	+44	+49	+33	+16	−6	−20	0	0	−3	0
280....	+6	+10	+8	+6	+9	+19	+26	+32	+36	+40	+45	+35	−2	+4	0	0	0
290....	+6	+11	+9	+8	+14	+31	+30	+25	+26	+40	+52	+40	+12	+10	+10	+3	+5
300....	+6	+11	+10	+9	+20	+35	+25	+15	+23	+35	+50	+45	+30	+25	+14	+10	+10
310....	+6	+12	+11	+10	+18	+40	+43	+36	+30	+30	+38	+50	+37	+30	+19	+15	+11
320....	+6	+12	+12	+10	+8	+14	+26	+34	+42	+46	+52	+60	+40	+29	+23	+18	+11
330....	+6	+13	+12	+6	0	0	+14	+17	+26	+40	+53	+62	+40	+28	+14	+16	+10
340....	+7	+13	+14	+11	+13	+10	+5	+7	+10	+24	+38	+64	+47	+30	+20	+14	+8
350....	+8	+14	+16	+20	+21	+9	+2	−15	−30	−10	+10	+56	+40	+24	+16	+12	+7

in all latitudes, and is conspicuous to the unaided eye in the great richness of the star clouds in the region Cygnus-Sagittarius-Crux, and the relatively low density in the remaining half of the Milky Way. It is easily traced on Figures 4–8 and in the curves of Figures 9–13 and, next to the galactic concentration, is the most striking characteristic of the general distribution of the stars. It is without doubt a reflection of the eccentric position of the sun within the galactic system, and, as here revealed, is a more complete and general exhibition of the evidence which has led earlier investigators to locate the center of the galactic system in the general direction of longitude 325°.

The curves of Figures 9–13 suggest the possibility of representing the greater part of the deviations collected in Tables XV*a–e* by the cosine curve

$$\Delta = a + b \cos (\lambda - L') \ .$$

Values of a, b, and L' have been calculated by least squares for each latitude and each limiting magnitude. For points of equal weight, equally spaced in longitude, the solution is very simple; and to retain this simplicity the interpolated deviations for $m=9$ and 11, which are inclosed in parentheses, were included in the solution. Certain deviations for $m=16.0$ and 18.0 which fall in regions of heavy obscuration are so clearly abnormal that they have been rejected. The most important of these are associated with the area of low density between the two branches of the Milky Way, which extends down to and includes the obscuration in Ophiuchus. The densest parts of the Ophiuchus cloud, and of the Taurus cloud also, are not much in evidence in the curves, because they lie between the Selected Area fields. The rejected deviations are:

	Latitudes	Longitude
For $m=16.0$......	+30°, +20°	320°–10°
	+10, + 5	330 –20
	− 5, −10	340 –30
	−20	130 –150
For $m=18.0$......	+20 to −10	330 –20

The results for a, b, and L' are collected in Table XVI and illustrated in Figures 14–16.

TABLE XVI

PARAMETERS FOR COSINE FORMULA REPRESENTING VARIATION OF DEVIATIONS WITH LONGITUDE

GAL. LAT.	$m=9$			$m=11$			$m=13.5$			$m=16$			$m=18$		
	a	b	L'	a	b	L'	a	b	L'	a	b	L'	a	b	L'
+70°...	−0.033	0.021	169°	−0.044	0.026	204°	−0.031	0.080	106°	−0.033	0.036	271°	+0.008	0.094	340°
+60....	− .077	.089	254	− .057	.055	247	− .042	.042	135	− .027	.045	299	+ .003	.138	322
+50....	− .091	.086	268	− .080	.077	258	− .057	.049	205	− .011	.093	336	+ .009	.142	336
+40....	− .126	.097	274	− .088	.082	258	− .047	.046	227	− .020	.068	334	− .013	.139	331
+30....	− .113	.111	273	− .082	.097	259	− .027	.063	260	+ .015	.125	307	− .007	.133	311
+20....	− .109	.128	255	− .074	.104	248	− .018	.073	239	+ .058	.188	301	+ .056	.201	309
+10....	− .044	.071	250	− .075	.148	255	− .074	.205	249	+ .082	.230	298	+ .089	.272	308
+ 5....	....	....	......	− .066	.171	261	− .074	.243	257	+ .080	.292	301	+ .096	.302	302
0....	− .014	.184	266	− .042	.222	272	− .019	.197	264	+ .085	.347	303	+ .098	.340	301
− 5....	....	....	......	− .036	.257	276	+ .012	.229	279	+ .091	.360	317	+ .099	.357	313
−10....	+ .006	.331	271	− .024	.288	280	+ .045	.263	292	+ .115	.406	328	+ .109	.388	325
−20....	+ .027	.254	266	+ .010	.307	279	+ .072	.296	300	+ .070	.316	328	+ .056	.392	330
−30....	+ .034	.220	267	+ .019	.283	279	+ .049	.281	299	+ .009	.262	332	+ .001	.277	333
−40....	+ .026	.213	273	+ .025	.227	278	+ .032	.225	296	− .010	.192	331	− .014	.229	332
−50....	+ .030	.166	272	+ .013	.145	279	+ .033	.173	287	− .010	.126	345	− .017	.166	343
−60....	+ .059	.126	272	+ .038	.094	276	+ .066	.081	279	.000	.073	354	+ .006	.092	353
−70....	+0.114	0.074	263	+0.074	0.049	263	+0.082	0.030	276	−0.027	0.080	340	−0.012	0.062	331

One-half the sum of the values of a for equal latitudes north and south represents the departure of the adopted mean distribution in Table XIV from the observed mean distribution as defined by the data now available. The mean values of a in the upper half of Table XVII are illustrated in Figure 14. The small negative correction shown for $m=9$ and 11 is too uncertain to be trustworthy. Otherwise, the agreement is satisfactory, except in latitudes 0°–20°, $m=16$ and 18. The correction thus indicated, which attains a maximum

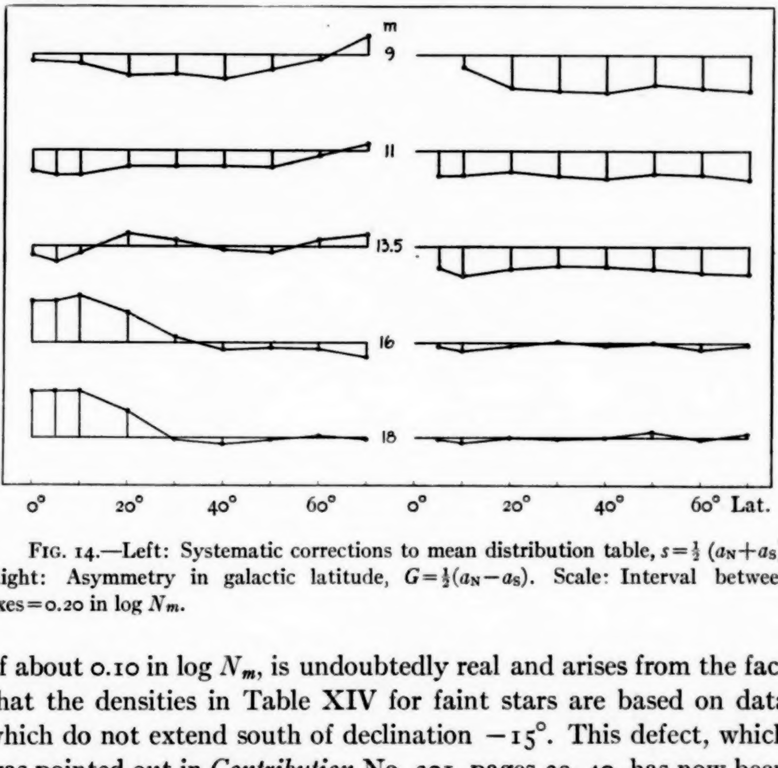

FIG. 14.—Left: Systematic corrections to mean distribution table, $s=\frac{1}{2}(a_N+a_S)$. Right: Asymmetry in galactic latitude, $G=\frac{1}{2}(a_N-a_S)$. Scale: Interval between axes = 0.20 in log N_m.

of about 0.10 in log N_m, is undoubtedly real and arises from the fact that the densities in Table XIV for faint stars are based on data which do not extend south of declination −15°. This defect, which was pointed out in *Contribution* No. 301, pages 29, 40, has now been partially removed with the aid of southern stars from the *Harvard-Groningen Durchmusterung*.

The difference in the values of a for equal latitudes north and south is a measure of the asymmetry of stellar distribution in galactic latitude. The excess of density in the southern hemisphere indicated

by the persistent negative differences in this parameter (see Table XVII and Fig. 14) for $m=9.0$, 11.0, and 13.5 is probably real; but its numerical value should not be too greatly stressed, since it depends on the systematic corrections in Table VII, which, as repeatedly stated, cannot yet be regarded as established for the brighter stars.

TABLE XVII

SYSTEMATIC CORRECTIONS TO TABLE XIV AND ASYMMETRY IN LATITUDE

Lat.	9	11	13.5	16	18
	Syst. Corr. $=\frac{1}{2}(a_N+a_S)=s$				
70°	+0.040	+0.015	+0.026	−0.030	−0.002
60	− .009	− .010	+ .012	− .014	+ .004
50	− .030	− .034	− .012	− .010	− .004
40	− .050	− .032	− .008	− .015	− .014
30	− .040	− .032	+ .011	+ .012	− .003
20	− .041	− .032	+ .027	+ .064	+ .056
10	− .019	− .050	− .014	+ .098	+ .099
5	....	− .051	− .031	+ .086	+ .098
0	−0.014	−0.042	−0.019	+0.085	+0.098
	Asymmetry $=\frac{1}{2}(a_N-a_S)=G$				
70°	−0.074	−0.059	−0.056	−0.003	+0.010
60	− .068	− .048	− .054	− .014	− .003
50	− .060	− .046	− .045	.000	+ .013
40	− .076	− .056	− .040	− .005	.000
30	− .074	− .050	− .038	+ .003	− .003
20	− .068	− .042	− .045	− .006	.000
10	−0.025	− .050	− .060	− .016	− .010
5	....	−0.051	−0.043	−0.006	−0.002
0	....	....	....	....	....

For $m=16$ and 18, on the other hand, the asymmetry may be regarded as sensibly zero.

Except for high northern latitudes, the values of L', the longitude of maximum deviation, are in rather striking agreement (Fig. 15). The amplitudes b decrease gradually with increasing latitude, and for any given latitude, north and south, are of the same general order of magnitude.

Qualitatively, these features are all explainable on the assumption that the sun is displaced from the center of a stellar system

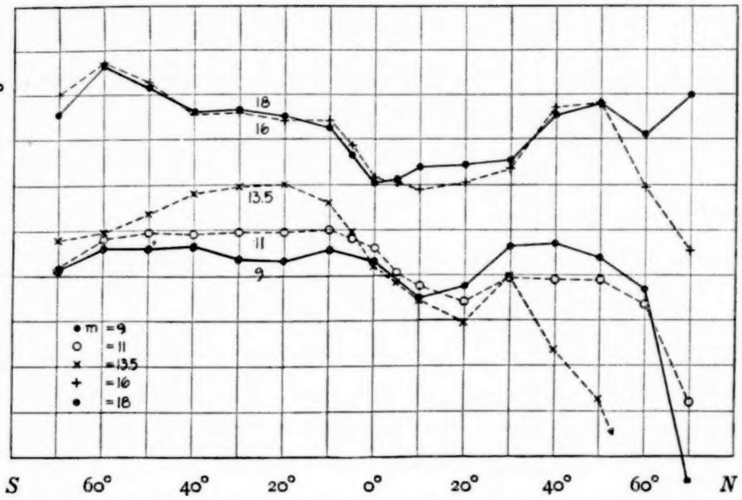

FIG. 15.—Values of L', the provisional longitude of the center of the stellar system, as derived from data to different limiting magnitudes in different zones of galactic latitude.

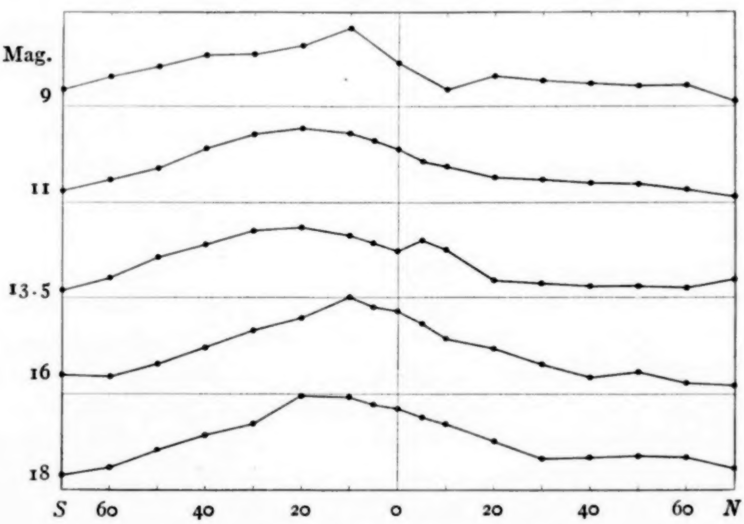

FIG. 16.—Values of b, the amplitude of the adopted corrections. Scale: Interval between axes = 0.40 in log N_m.

having spheroidal symmetry. Moreover, since the asymmetry of distribution in latitude is small, and even negligible for the fainter limiting magnitudes, the displacement must be very nearly in the galactic plane.

Closer examination of Table XVI shows, however, additional features that are suggestive. Were the conditions exactly those of the simple hypothesis just outlined, the longitude L' would be constant; further, since the solar system is certainly near the galactic plane, the values of the amplitudes b would be sensibly equal for the same latitudes, north and south. Systematic departures are to be noted, however, in both particulars. For example, L' depends on the limiting magnitude m; but, more important for the moment, it is systematically smaller, for all magnitudes, in northern latitudes than in southern. Some of the difference can be traced to irregular fluctuations in density, which are especially large in low latitudes. The influence of the great aggregations of the Milky Way for the present is regarded as accidental; but sometimes it is of such magnitude as to produce a systematic effect. Thus the minimum between 0° and 30° of north latitude shown by the longitude curves of Figure 15 is partly accounted for by the secondary maximum appearing near longitudes 180°–200° in the corresponding latitude curves of Figures 9–13. This maximum arises from the Monoceros cloud, which raises the density curve and displaces the adjacent nodal point in the direction of smaller longitudes. Such circumstances do not, however, account for all of the systematic difference between the two hemispheres.

The values of b also show a systematic effect, the maximum for each magnitude limit occurring in latitudes 10°–20° south (Fig. 16). At first sight, this might be attributed to the local influence of the great star clouds in Sagittarius and Scorpius, which have their centers south of the adopted galactic plane; but further examination indicates that some of this difference, as well as of that appearing in the longitudes L', arises from an error in the position used for the galactic pole (Gould's). The data are further analyzed from this standpoint in the following *Contribution*, No. 347.

Tables XIV and XV together provide new values of the stellar distribution to various limiting magnitudes in different parts of the

sky. Tables XV*a–e*, it will be observed, include any systematic correction required by Table XIV, and, at the same time, all deviations from the mean distribution falling within the limits of the fields actually counted, modified, however, by the averaging and smoothing used in preparing the tables. In and near heavily obscured regions and at points of great local concentration, the representation is still far from satisfactory; but it is, nevertheless, much more complete than that afforded by the mean spheroidal distribution which wholly disregards the conspicuous variations of density with longitude.

Since Table XIV is based on counts made in all longitudes, it is little influenced by an error in the adopted position of the galactic pole. The observed density at a longitude λ has, perhaps, been assigned to an incorrect latitude β, instead of to the true latitude $\beta_1 = \beta + \delta\beta$; but in longitude $\lambda + 180°$, the density assigned to latitude β refers sensibly to the true latitude $\beta_2 = \beta - \delta\beta$. The tabular density therefore corresponds very nearly to the mean of the true latitudes, which equals the adopted latitude β itself. For any probable value of the error in the position of the pole, the residual error in the mean distribution table will be vanishingly small, and Table XIV may be used as it stands; it is only necessary that correct latitudes be employed. This remark applies only to the determination of mean densities referred to correct latitudes.

Although error in the position of the pole is thus eliminated from Table XIV, there is no such elimination from Tables XV*a–e*, and co-ordinates referred to Gould's pole must be employed in using these tables. Thus, if we wish to find the limiting magnitude corresponding to a count of stars at a certain point in the sky, Tables XV*a–e* must be entered with co-ordinates referred to Gould's pole. Strictly speaking, an approximate limiting magnitude, for use in connection with Tables XV*a–e*, must first be found from Table XIV with the aid of the observed density. The correction thus derived from Tables XV*a–e* is to be subtracted from the observed density, which is thereby reduced to the mean distribution. The result is then used to interpolate the final limiting magnitude from Table XIV. The adopted latitude must also be used for this operation, because Tables XV*a–e* were formed by comparing observed densities with

mean densities interpolated from Table XIV with latitudes referred to the adopted pole. Similarly, in the inverse problem of finding the probable density corresponding to limiting magnitude *m* at a certain point in the sky, the co-ordinates referred to the adopted pole must be used in connection with both Tables XIV and XV*a–e*. Technically, the procedure outlined is correct; but in most cases the gain in precision will be less than the inherent uncertainty of the tables.

Carnegie Institution of Washington
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December 1926

REVIEWS

The Internal Constitution of the Stars. By A. S. Eddington. Cambridge University Press, 1926. Pp. viii+407. Figs. 5. 25*s*.

This important book affords the first systematic account of the most remarkable field of advance in modern astrophysics—presented by the investigator who has led in the advance.

Fifty years and more ago, it was known that the pressure at the sun's center must be thousands of times greater than any which we could hope to produce artificially, and that the same must be true of the temperature, provided that the properties of matter could safely be extrapolated so far beyond the range of observation. It is only within the last dozen years that we have had any evidence that the extrapolation was to be trusted; now we know how atoms behave when abused even more violently than they are likely to be in the stars, and have a safe foundation to build upon. We find our problem greatly simplified. The material inside of a star is so dissociated and ionized that we have to deal only with a practically "perfect" gas, whose principal properties are almost independent even of its chemical composition. Just as the motions of the planets present (in their main outline) the simplest of the great natural problems of dynamics, the constitution of the stars appears to offer the simplest of the great natural problems of physics. In its main lines, it is already far toward solution; the finer details bid fair (as in the case of the planets) to afford plenty of room for long-continued hard work.

In the present volume, Professor Eddington has done much more than to collect the scattered literature of the subject (up to the date of completion, early in 1926). He has worked it into a consistent whole, has replaced many difficult discussions by much simpler ones, and has produced a remarkably lucid book. To accuracy and intelligibility is added his characteristic brilliancy of style. Space allows only a single quotation (p. 23): "When aether waves fall on an atom they are not absorbed continuously. The atom lies quiet waiting a favorable chance and then suddenly swallows a whole quantum at once. The mouthful is too big for the atom's digestion; consequently the atom bursts." Among the longer passages, two deserve special mention—the vivid description of atomic events inside a star (pp. 19–20), and, above all, the admirable discussion

(pp. 101–103) of the differences between the aims and methods of the mathematician and the physicist. All students of physics may well "read, mark and inwardly digest" this passage, and the reviewer concurs fully with its conclusion: "Cases could be cited where physicists have been led astray through inattention to mathematical rigour; but these are rare compared with the mathematicians' misadventures through lack of physical insight."

The book begins with a "survey of the problem," which may be read to great advantage by the elementary student. Then come two admirable chapters on the thermodynamics of radiation and the quantum theory, which contain just what the serious student of astrophysics needs to know. Considering the subjects, these chapters are very easy reading. The discussion of the main problem begins with a chapter on polytropic gas spheres, closely following Emden. The reviewer pauses here to note how great an influence on the development of the theory of stellar constitution has been exerted by Emden's convenient set of particular solutions of the problem of internal equilibrium. Practically all later workers have made the approximations which have usually been unavoidable in such a manner as to bring in these flexible and already tabulated functions. The very heavy labor of calculating new sets of solutions justifies their choice; but it should be remembered that, so far as is known, the polytropic functions have no such natural relation to the problem as spherical harmonics and Bessel functions have in their own fields. Functions better adapted to the present problem may some day be invented.

In chapter v the equations of radiative equilibrium are set up. Rosseland's distinction between absorption and opacity is taken into account, but detailed discussion of certain terms of the order 10^{-18} is "left to the reader." Chapter vi applies these relations to the interior of a star. The equations are greatly simplified if a certain quantity ηk is constant (k being the coefficient of opacity, and η the average rate of generation of heat per unit mass in a given part of the star). This assumption leads to a star of a definite polytropic constitution, in which the ratio $1-\beta$ of radiation pressure to total pressure is everywhere the same. The relations connecting the mass, radius, and molecular weight with the luminosity and the central and effective surface temperatures are derived for stars built on this model. A detailed discussion of the observational evidence, which is in striking agreement with the theoretical mass-luminosity curve, occupies most of the next chapter.

This agreement marks the theory of stellar luminosity as one of the most brilliant successes of modern physics. Indeed, this success went be-

yond its author's anticipations, extending to the dwarf stars as well as the giants, and solving the apparently hopeless problem of the white dwarfs. The reasons, both theoretical and observational, for believing that these stars are thousands of times denser than platinum are fully and clearly set forth. The one outstanding difficulty—the question of the final state of matter when so dense a star cools down—has been removed by R. H. Fowler since the book appeared.

Good as this whole discussion is, the reviewer feels that rather too much prominence has been given to the particular "model" already mentioned. Whether or not this is a good approximation to the facts depends upon the form of the law of generation of heat inside the star, which is still almost unknown. A slavish adherence to Eddington's model would demand that an outer shell of the star, containing about one-fifth of its mass, should act continually as a "sink" of heat, which was absorbed and converted into some form of potential energy. Though one would no longer venture to call this conclusion physically absurd, it appears to be highly improbable. It is much more likely that the outer regions neither generate nor absorb heat, but simply allow the flux from the deeper parts to pass through.

This difficulty is not explicitly mentioned by Eddington, but is fully met by earlier remarks (pp. 95, 116) which show that the existence of such an inert outer layer would not seriously affect the mass-luminosity relation. Moreover, he has shown, even in the extreme case when all the heat is generated at one point in the center, that the luminosity of a star of given mass, compared with one following his model, will be decreased by only one stellar magnitude. This computation, which involved very laborious quadratures, was made for but one value of the mass; but there is no doubt that the effect for other masses would be of the same order of magnitude.

It is not hard to prove, indeed, that a mass-luminosity relation of the same general form would be found for stars built on any other definite model, and that, for a given mass, very considerable changes in this model would be required to affect the absolute magnitude seriously. The validity of Eddington's theory of stellar luminosity is therefore practically independent of the accuracy of the particular model which he adopts as convenient for analytical discussion. These conclusions may be extracted from the book itself by a careful reader (for example, in the second footnote on p. 134); but the point is so important that it might well have been given a more prominent place.

Chapter viii deals with variable stars, and mainly with the pulsation

theory of Cepheids. The rather difficult analytical work is clearly presented, and a judicious use is made of arguments based on the conditions in a typical region of a star, which suffice to determine the order of magnitude of the quantities sought (such as the damping of the oscillations) without the heavy labor of detailed quadratures. The application of this theory to actual Cepheids is doubtful since Eddington's recent work[1] has shown that the serious discrepancy in phase between the observed and computed changes of temperature cannot be removed by anything that happens in the outer layers of the star. The pulsation theory, however, is still valuable in the discussion of the internal stability of the stars. The condition of "overstability"—as Eddington calls it—in which a pulsation, once started, would increase to large amplitude, cannot be predicated of ordinary stars. If it does not occur in Cepheids, either, its importance as a criterion by which to test theories of stellar constitution is enhanced.

Eclipsing variables are briefly discussed—mainly as regards the "reflection" effect. Here it may be remarked that some of the stars for which the discordance between observation and theory is large (RT Persei, U Cephei) have been exceptionally well observed, so that the discrepancy cannot be attributed to observational error. There are also many eclipsing variables such as RT Lacertae, RZ Ophiuchi, W Crucis, which are conspicuously giant stars and must have low central temperatures so that the question whether such close pairs are formed only when the central temperature is 40,000,000° is to be answered in the negative.

Chapter ix deals with the coefficient of opacity. After discussing the work of Kramers and others, the relation

$$k \propto \frac{\rho}{\mu T^{\frac{7}{2}}}$$

is adopted as the law "or something sufficiently near it for most stellar applications." The tenfold excess of the "astronomical" above the theoretical value remains unexplained. The following chapter discusses ionization, diffusion, and rotation. The results of Fowler and Guggenheim regarding the first are accepted. Diffusion, if proceeding to the limit, would concentrate light and heavy atoms in different parts of a star, to a degree incompatible with observation; but this process should require something like 10^{16} years—far longer than the probable life of a star. The production of vertical convection currents as a result of the uniform rotation of a star is accepted as a consequence of von Zeipel's theorem, and regarded as sufficient to counteract any separation by diffusion. Jeans's

[1] *Monthly Notices of the Royal Astronomical Society*, **87**, 539, 1927.

remarkable proof that the rotation of the inner parts of a star must be much more rapid than that of the surface came too late for inclusion.

The source of stellar energy is discussed in chapter xi. It is concluded that the rate of liberation of subatomic energy must increase with the temperature, and perhaps with the density, and that, under these conditions, a star would be stable, though too rapid a rate of increase would result in overstability.

The nature of the process involved is not defined beyond saying that the source must ultimately be exhaustible—though the transmutation of hydrogen into other elements, and the complete conversion of matter into energy are favorably considered.

The difficulties in which any such hypothesis becomes involved are instructively stated. The astronomical difficulties appear less serious to the reviewer than to the author. In particular, the conclusion that giant and dwarf stars in such a cluster as the Hyades cannot be of the same age is not a necessary consequence of the theory, and this objection disappears. The physical difficulty that it is very hard to see how temperatures less than 100,000,000° could have any influence on processes liberating so much energy is frankly stated. In view of the other evidence that it actually happens, these considerations are met with a sentence which should become classic: "We do not argue with the critic who urges that the stars are not hot enough for this purpose; we tell him to go and find *a hotter place.*"

Two concluding chapters deal with topics cognate to the main subject. "The Outside of a Star" follows the trails blazed by Milne, and deals rather fully with photospheric problems. The problems of "absorption" lines and of the chromosphere are touched only on the surface. The opinion (p. 353) that the broadening of stellar and solar lines may be due largely to "pressure" effects appears to be hardly tenable, in view of the fact that those lines which are most widened by pressure in the laboratory are sharp in the sun.

The final chapter, "Diffuse Matter in Space," develops the author's theory of the origin of the "fixed" lines of Ca^+ and Na, which appear in the hottest stars. Whether Jeans's recent criticism will lead to any change in these views remains to be seen. The discussion of general absorption of light in space, and of dark nebulae, is influenced by "the impression (perhaps not too well founded) that the primordial state of matter must be gaseous," and that meteors or meteor dust must be the débris of some former aggregation of matter" (p. 386). The reviewer does not share this impression, nor would he be surer of its opposite. The fact that, for equal

density of distribution in space, fine dust is enormously the most effective obscuring agent, appears to be more important than any a priori impression. The allied fact that such dust, if smaller than the wave-length, will produce selective scattering, following Rayleigh's law, but vastly greater per unit mass than in the case of a gas, is not mentioned.

The duty imposed by custom on a review, of pointing out errors, is in this case remarkably difficult. One or two misprints appear to have crept into equations, but in no case are the errors carried through into the following work. The only serious criticism which might be made is that too little attention is paid to the extensive work of Jeans. In the reviewer's judgment, Eddington is in the right in most of the matters upon which these distinguished investigators differ, and the arguments which he gives unobtrusively here and there on these points are convincing; but a fuller treatment, both of these points and of others not yet settled, such as the choice of a different approximate model for the internal constitution of a star, would have been of advantage to the reader, and have added to the value of the present book.

In fine, the book is one of quite unusual importance and value, and no astronomical or physical library, nor any serious student of the subject, can afford to be without it. In appearance and typography it conforms to the high standard which one expects from the Cambridge University Press.

HENRY NORRIS RUSSELL